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RECENT HYDRAULIC EXPERIMENTS.

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Minutes of Proceedings of the Institution of Civil Engineers.

II.

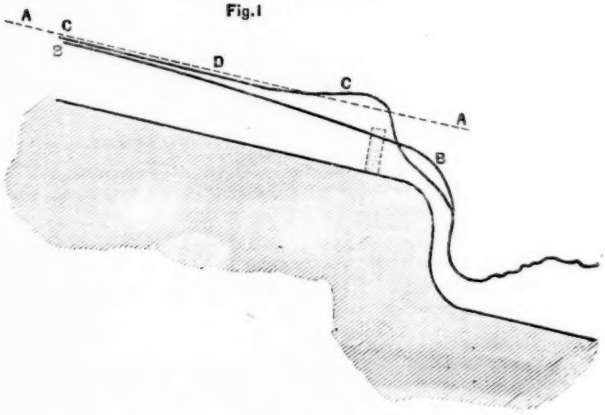
Discussion.

Major Allan Cunningham desired to add that the experiments, of which an account had been read in abstract, were very extensive, the complete report of them occupying three large volumes. It had been difficult to prepare a condensed report of the work which should give a fair account of it, and at the same time not be tedious to the meeting. The subject of the flow of water was an important one, both scientifically and financially. It had been stated that the crops which were saved in a single season of drought by irrigation from the Ganges canal, on which the experiments were made, yielded a revenue which covered the entire cost of the construction of the canal—a revenue which otherwise would have been entirely lost. The subject was a dry one; but Figs. 6 and 7 showed two average curves which might be taken as a summing-up of the whole of the experiments.

Professor W. C. Unwin fully acknowledged the credit due to the author for the labor he had expended on his experimental research, and for the high standard of accuracy and carefulness which he had aimed at throughout. There was no point of detail to which he had not paid attention, or in which any one could say that he had been careless in his observations. He hoped that in speaking on two or three points, in regard to which he differed somewhat from the author's conclusions, it would not be thought that he differed in regard to the principal points that had been worked out, or with Major Cunningham's general conclusions. He should like first to call attention to one point in the experiments, which he thought explained why it was that the author's measurements of discharge did not throw any great light on the relative value of the different formulas in use. The Ganges canal, as originally constructed, was in reaches of uniform slope, with a sudden drop at the end of the reach. It seemed to have been supposed by the constructors of the canal that the water would flow as it did in an ordinary stream of uniform depth, as at AA, Fig. 1, and that there would be a sudden drop at the fall. But it was well known that a sudden drop in a stream bed produced a form of water surface such as BB. The effect of an oversight, as it appeared to him, as to the influence of the sudden drop in the bed of the falls, was that the velocity in the canal was greater than was expected. At all events, the scouring action was greater,

and that was corrected by building at the falls temporary weirs, which backed up the water and gave the water surface a shape like CC. At some of the sites where the largest number of experiments were made, Major Cunningham had placed himself at the point D of the canal, where obviously the surface-slope was most variable. The case was still further complicated by the fact that, in regulating the discharge of the canal, temporary obstructions were placed at the falls, and that, in low-water conditions of the canal, the obstruction at the fall was higher than the bed of the canal at the site of the gaugings. From one point of view that was extremely interesting. The author had shown that the discharge under those conditions bore no relation to the depth of water in the canal, and he had thrown light in various ways on the action of water in a reach of that kind. But Professor Unwin considered that a site of that kind was not suitable for independent observations to verify the formulas of discharge; and the confession in the paper that the measurement of the surface-slope was an exceedingly uncertain measurement, seemed to admit that the site of the gaugings was inconvenient for that particular purpose. It had been mentioned that the surface-slope had been measured in lengths of 4,000 feet and 2,000 feet, and that there was a difference of 25 per cent. Of course, that threw great doubt on what slope ought to be taken in calculating discharges, by any formula which involved the use of the surface-slope. Gaugings were made over a length of 50 feet only, and it was still further uncertain whether the slope over 50-feet length was anything like the slope over 2,000-feet length. It seemed to him that the measurement of the surface-slope over a great length, when the gauging was over a very short length, was not a satisfactory proceeding. If the surface-slope was measured by leveling operations, it was essential to measure over a considerable length. Of course, a leveling instrument was not accurate enough to measure the surface-slope in a short distance; but it did not seem impossible to adopt micrometric methods in measuring the slope in a very short distance. It would be possible to have a baulk of timber, say 40 feet in length, of sufficient weight and solidity to hold a tolerably stable position in the water, on which might be placed a level with micrometric screw, so as to measure the slope, say in a length of 40 feet, to $\frac{1}{1000}$ inch. In that case it would be possible to get the surface-slope precisely at the site of the gauging.

Fig. 1



All the experiments had been made with floats, and some discredit had been thrown on the use of current-meters for gauging. He did not say that one was better than the other, because there were cases in which floats must be used and current-meters could not be used; but, as he had lately had a good deal of ex-

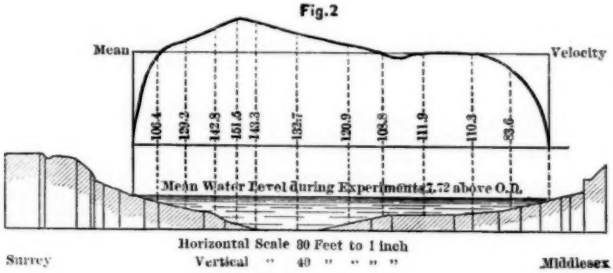
perience in the use of current-meters, he would say a few words as to the relative value of the two kinds of instruments. The author had enumerated several advantages of floats: 1st. "They interfere little with the natural motion of the water." That was true, and he would pass it by. 2d, "they measure velocity directly." It was true that they measured the average velocity over a more or less considerable length of stream, but not at the section point; he did not think that was of much importance. 3d, "they can be used in streams of any size." 4th, "they are not much affected by silt or floating weeds," which was quite true. 5th, "they measure forward-velocity," which was also true. 6th, "they can be made up and repaired by common workmen," which did not seem to be very important. And 7th, "they are very cheap." No doubt floats could be made very cheaply, but if, in the observation of velocity, they required two or three times the time which current-meters required, the instrument was not really a cheap one. In determining each velocity, the author had taken forty-eight float observations, over a length of 50 feet; in other words, he had practically observed the average velocity over a length of 2,500 feet. He should show presently that with a much shorter length of stream current-meters gave practically a constant velocity. There were certain disadvantages in the use of floats. In the first place, it was impossible to get rid entirely of the action of the wind on the exposed surface of the floats. Although surface-floats were, of all others, the least open to objection, it was just the surface-velocity which would be the last that would be observed if velocity could be recorded at all points of the section with equal facility. The moment it was attempted to get velocity below the surface by the use of floats, difficulties were encountered which were considerable. Using what appeared to be the best instrument, according to Major Cunningham, the sub-surface float, with a very light surface-float, it was impossible to get rid of the stream on the cord which connected the surface-float with the bottom-float, and that almost restricted the use of sub-surface floats, for anything like accurate measurements, to very moderate depths of water. When it was remembered that, in the Mississippi experiments, the cord connecting the surface and the bottom-float had an area one and a half time as great as the area of the sub-surface float, it would be seen that, neglecting the influence of the cord must, in that case, have introduced an enormous amount of error. On the other hand, the current-meter, supposing it to be rightly fixed, and of one of the forms which obviated the objections to engaging and disengaging gear, certainly enabled velocities to be taken with more rapidity. He took some velocities in the tidal part of the Thames in January, 1882, and on the average of five or six days' work, he obtained one good observation in about three minutes' work, a rate which he thought could not be approached in using floats. Then the velocity was obtained at the section at which it was wanted. Finally, it should be remembered that, with floats, unless the stream was comparatively narrow, there was the laborious operation of fixing the float-path by angular measure. There was one point with respect to which he thought there was a little misconception. Looking at the face of the current-meter, the blades of the screw appeared very large, and seemed as if they would interfere a good deal with the motion of the stream; but when the instrument was at work it was only the edges of the blades that met the stream, and he had a strong impression that the current-meter interfered very little with the natural action of the water, if only it was made with the right degree of delicacy. He had been using a current-meter which had a very convenient arrangement — not being fixed on any rod, but suspended by a wire. It was only open to one kind of objection, so far as he knew, that when the meter was suspended by a wire, it was uncertain whether it was exactly normal to the plane of the section, the mean velocity across which it was intended to measure; there was, therefore, a source of error in that kind of suspension, but it was as well to consider what magnitude that error was likely to assume. Suppose a meter to be fixed at 20° with the normal to the section the velocity measured would be erroneous by about 6 per cent. Now a meter suspended by a wire was not fixed at an angle with the plane of

section, but it swayed backwards and forwards through a more or less large angle. Supposing it to sway backwards and forwards, through an angle of 40°, the error of observation due to its position would not exceed 3 per cent. That, however, he believed was a much greater angle than any through which the meter swung, and the error did not seem to be a large one; so that, at all events for many purposes, a meter suspended by a wire was an extremely convenient form of current-meter. With respect to the unsteadiness of the motion of water in a stream, he had used a meter during the last flood in the Thames in this way; he took a continuous note of the time of each hundred revolutions of the meter, and he had plotted in the three curves shown on the diagram (Fig. 8) the results of the observations taken in that way at depths of 0.5, 3, and 6 meters from the surface. Taking every hundred revolutions, there was an exceedingly irregular curve. The oscillations of the velocity during periods of twelve seconds for the two upper curves, and twenty seconds for the lower curve, were very large. But by simply averaging the result for five hundred revolutions of the meter instead of one hundred, there was an exceedingly regular curve. There were eleven or twelve successive periods during which the meter made five hundred revolutions. The velocity at the end of each interval was calculated from the five hundred revolutions, and a curve was obtained which approached closely to a straight line, and he was not sure that the irregularity which existed was not due to an imperfection in observing the time. He hoped to be able to repeat the observations with an electric chronograph, which would eliminate all error of that kind. One point in the paper had not been alluded to—a point on which he entirely differed from the author. There was a well-known phenomenon of flowing streams that the position of the line of maximum velocity was considerably below the surface, often at about one-third of the depth at the middle of the stream, more or less towards the sides. The explanation of that depression of the line of maximum velocity had been considerably discussed. The explanation adopted by the author was, that the air opposed a resisting surface against which the water rubbed. The retardation due to the friction of the water against the air explained, he thought, the reduction of surface-velocity and, therefore, the depression of the line of maximum velocity. Although Professor Unwin did not deny that the air retarded the motion of the water, the explanation appeared to him to be an altogether inadequate one. The only cause which seemed to him sufficient was the mixing with the surface water of water stilled by contact with the bed, and brought up to the surface. There was more than one way in which that occurred. Eddying masses of water produced against the roughness of the bed were shot off and mingled with the stream, and were liable to accumulate at the surface because it was a boundary of the section. Probably, in addition to that, even in a straight length of stream, a curvilinear motion of the water in spirals brought up the bottom water towards the surface; and furthermore at every bend in the river there was demonstrably a rotation of the water in the plane of the transverse section. Amongst the most interesting results mentioned were the author's attempts to find some rapid way of approximating to the mean velocities in the section of the stream; and he had given two or three rules for finding the mean velocites of a stream, from observations made at two depths at the center of the stream, or by one observation with a peculiar float which had a small surface float and two equal sub-surface floats. In finding that means of rapid approximation the author had proceeded entirely in one direction; he had tried to find the mean velocity from observations at two different depths. But Professor Unwin thought the mean velocity might be more easily found by observation at two positions in the horizontal width. To test this he had worked out as a rough trial two considerable sets of the author's observations made with rod-floats, and he had found that if single observations had been taken with a rod-float at very nearly one-third of the breadth of the stream from the center, he would have got approximately the mean velocity of the stream, and much more accurately if he had made two observations with a rod-float at one-third the distance from

the center on each side of the center line. He should like to mention one point which it appeared those who measured the flow of the water had been a little too apt to neglect. So far as he knew there were no observations on flowing streams in which the temperature of the had been observed. He had found in experiments made in another way that, at the temperature of the atmosphere, about 60°, a very few degrees difference in temperature made a marked and measurable difference in the fluid friction; and he thought that in future it would be useful if experimenters would record the temperature of the water at the time their observations were made. He believed that some noticeable discrepancies in the results might perhaps be explained by observing the temperature. He had placed on the wall the results of two gaugings which he had made during the last year. One was a diagram of gaugings during the last Thames flood. He had drawn a curve of mean velocity which looked tolerably regular, and from it calculated the flood discharge. It was a little under 8,000 cubic feet per second—a result not very different from that which he had given to the Institution some years ago. The flood was six inches lower than that of 1875, and at the section which he had chosen this year for gauging a large quantity of water, 3,000 or 4,000 cubic feet possibly, was escaping over some miles of flooded area on the bank of the river. He exhibited some results of velocity observations at a section of the river at Putney, in order to show how far it was possible, in a tidal stream in which the surface slope and velocity varied from instant to instant, to get enough observations to calculate the discharge of the stream. The flow over Teddington Weir in the flood while the observations were being made was just under 71 cubic meters per second, or 2,500 feet per second.

Mr. Baldwin Latham exhibited a diagram, Fig. 2. showing the result of a number of gaugings which had recently been made in the River Thames, immediately below Teddington Lock. The gaugings were all made about the period of low water, after the tidal water had run off, and therefore represented the flow of upland water of the Thames at that point. At the place where the gaugings were made the river was slightly curved, the concave side of the curve being on the Middlesex side of the river, and the convex side on the Surrey side.

Fig.2



RIVER THAMES.

The ordinates above the section of the river represented the relative velocities at the points in the section of the river over which they were placed. The horizontal line represented the mean velocity of the stream. It would be seen that at one point there was a depression in the curve, showing that the velocity there was less than on each side. This occurred over a shelving bank on the concave side of the river; while, on a similar shelving bank on the convex side, there was an abnormally high velocity. There could be little doubt that these peculiarities were due to the horizontal deflection of the stream; for although the diagram was based upon upwards of 60,000 feet run of the current meter, extending over numerous observation, the separate observations all more or less indicated this distinctive feature. With reference to the use of double-floats in gauging, no doubt

under some circumstances they might be useful, especially in sluggish streams; but in the generality of streams the current-meter was undoubtedly preferable. Where the maximum velocity was removed one-third of the total depth of the stream from the surface, and where the velocity was represented by the ordinate at any point in the vertical section of a parabola having its axis parallel with the surface of the stream double-floats were not necessary, as the surface-velocity in that particular case would be equal to the mean velocity, and so a single surface-float would give the true velocity of the stream. Judging from his own observations, he should say that it was very rare indeed for the maximum velocity to be removed so far as one-third the total depth of the stream from the surface. Where such a condition did exist, it was no doubt due to the vertical deflection of the stream downwards, arising either from the channel being deeper at the place of observation than higher up the stream, or from water being tailed back over the point of gauging. This, as suggested by Professor Unwin, might have been the case at certain points in the canal which have been selected as gauging stations. The effect of the damming back was to flatten the inclination in the lower portion of the longitudinal section of the canal, and the water entering from the upper portion at an angle, naturally was projected downward, and so the maximum velocity was removed from the surface. It was important to determine the position of maximum velocity. Looking at Professor Unwin's diagrams it appeared that the mean velocity was not far from the center of the stream; but in most channels of trapezoidal cross-section, in which the larger volume of water was moving with a relatively higher velocity, the point of mean velocity in the stream would rise above half the depth. As in rivers and streams of pretty constant flow there were two points in every section where the water would be moving at the mean velocity, if these two points were ascertained a ready means would be available for easily gauging, by a current-meter, the quantity of water passing. He had made numerous gaugings in this way, but he took care at the same time to check the records by an instrument which delineated every change in the area of the section. Great precautions were needed in arriving at results based on calculations in which the use of coefficients, and the rate of inclination of the surface of the water, were the principal factors. It was well known that within certain limits the length of the channel affected the velocity of flow. In long channels of uniform grade, as compared with shorter channels there was an acceleration of flow; for example, the velocity of water in a 12-inch pipe, full or half full, having an inclination of 1 in 495, when the length was but 250 feet, would be 2 feet per second; but if the length were increased to 30,000 feet, the inclination remaining the same, the velocity would be $2\frac{1}{4}$ feet per second. He had found by actual measurement in the river Wandle that weeds attached to the bottom of the stream, and growing in the water-way of the channel, affected the flow of water to an enormous extent; for while the ordinary Eytelwein formula in the Chely form gave a coefficient of 93.4, in this case the coefficient, on an average of six carefully conducted experiments, fell to 40.23. This proved the enormous influence exercised by the growth of weeds in impeding the flow of water in a channel. With reference to the form of the curve at the surface of the water, there was no doubt that whenever a river was filling up, the surface of the water was convex, and that whenever the river was falling it was concave. If the stream remained constant there would be a pretty level surface. When a river was filled up two functions were in operation, the water was moving down the stream, and the channel was being filled. As water was moving down the stream the maximum flow was always in the deepest part, generally in the center of the stream. The consequence was that the water fell from the center towards the banks, but the reverse condition held when the stream was falling. In Mr. C. Ellet's work on "The Mississippi and Ohio Rivers," it was recorded (page 302) that whenever the latter river was rising the drift was thrown on the shores, clearly showing a current from the center outwards; but whenever the river was falling, the boatmen said they could travel for miles in the center of the stream without making a single sweep of the oar

to keep them in the current. This amounted to an actual demonstration of the curves. The author of the paper seemed to say that no results had been obtained from the observation upon silt. Mr. Latham presumed that that simply meant that a large quantity of silt was brought into the canal from day to day, and that the simple velocity of the stream itself had not much influence upon moving or creating the silt; but he wanted silt observations to be carried on from quite a different point of view. At Croydon there was a culvert 4 feet in diameter under the town. At certain periods it discharged beatifully clear spring chalk-water, at the rate of 3,000 to 4,000 cubic feet per minute. It was also connected with the street gullies of the town, and in time of rain discharged highly-colored water carrying a large amount of silt. He had been making careful observations on the culvert for many years, and he had found that whenever the water was clear the maximum velocity, with the same height and gradient, was always considerably higher than when the water was turbid; thus the mixture of silt with water had the effect of retarding its velocity. He considered that the retardation in this case was due to the extra load the water had to carry; and if the weight of water and weight of silt were taken and multiplied by the velocity, it would be found to tally pretty nearly with the weight of the water multiplied by its velocity when there was no silt. Thus by the addition of silt a diminution of velocity was effected, while in the absence of silt the velocity was greater. He contemplated making a series of observations with different percentages of material put into water, to ascertain to what extent the velocity of the water might diminish. However, it was obvious from observations already made, that silt, when present in a large quantity, did interfere with the velocity of flow. The author seemed much surprised that, with such a high temperature as 165° in the sun and 105° in the shade, there was only $\frac{1}{10}$ inch of evaporation per day from the surface water. For some years he had been carrying out experiments on evaporation. In former days it was considered that the evaporation in England greatly exceeded the rainfall, and there could be no doubt that the instrument used did show that such was the case: but common sense would show at once that if that were so there would be no streams. Mr. Charles Greaves, M. Inst., C.E., who had done so much to elucidate the question of evaporation, and other meteorological phenomena in which engineers were interested, had directed his attention to the views of Dalton on evaporation. These tended to show that evaporation was not due to temperature, but to the vapor-tension. Water at a particular temperature would give off vapor of a certain tension. The vapor in the air also had a certain tension, which was arrived at by the temperature of the dew point. So long as the tension of the vapor in the air was less than the tension of vapor due to the temperature of the water, water would pass into the air, but when the tension of the vapor in the air was greater than the tension of vapor due to the temperature of the water no evaporation would take place. Although the author of the paper recorded no experiments with the wet and the dry bulb thermometer, it was probable that at 165° and 105° the atmospheric vapor-tension would be in excess of the the tension of vapor of water at 66°. No one could expect to blow steam from a steam-boiler under 10 lbs. pressure into a boiler which was under 20 lbs. pressure; but it would be easy to pass the steam from a boiler under 20 lbs. pressure into a boiler under 10 lbs. pressure. That was exactly what was found in evaporation. The month in which the largest amount of evaporation had occurred this year was May, yet the temperature of May was much below that of July. He would point out a source of serious error in all the experiments on evaporation which he had tested. He had adopted as the standard form of evaporator the form introduced by Mr. Greaves, namely, a floating evaporator, but he originally started with one similar to that described by the author. He had a vessel floating in water. It was supported by an outside ring, which came above the water, but he found that when the sun in hot weather shone upon the evaporator, as the water always ran up the side of the vessel by capillary action, great evaporation occurred. The error in all such vessels was no doubt due to the film of water that ran up the

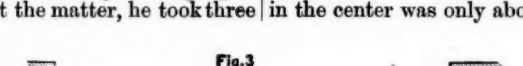
sides, and which under great heat was speedily evaporated. He therefore discarded that form of evaporator, and suspended the evaporating vessel from a floating ring like a life-buoy, and at once there was a large reduction in the amount of evaporation. The film of water now extended up the sides of the vessel both on the outside as well as the inside, and so they were kept cool. He, however, carried the experiment still further on a number of evaporators painted different colors. He observed that from a painted gauge, even if black, there was a less amount of evaporation than from a plain copper gauge. That was because capillary action was more energetic in the metal gauge than in the painted gauges. In order to test the matter, he took three evaporators made of copper, each 5 inches in diameter, and holding at least 1 foot depth of water. One of these he had slightly greased inside, and that and another copper evaporator were allowed to stand in a tank of water immersed to within 2½ inches of the top, while at the same time the third evaporator was freely exposed to the atmosphere. The evaporation from these gauges in the month of May, 1882, when compared with that of the floating gauge, 12 inches in diameter, and a gauge painted white, but immersed in water, were—

Floating gauge.	Gauge in air.	Gauge in water.	Gauge greased in wat'r.	Gauge painted white in wat'r.
Inches.	Inches.	Inches.	Inches.	Inches.
3.665	6.355	5.495	4.085	4.175

In the case of the gauge greased regularly, the coating was so slight that no oily matter ever appeared on the surface of the water. The size of the instrument had a marked influence on the amount of water evaporated, for the larger the vessel, in proportion to the area, the smaller was the marginal ring up which the water passed by capillarity, and from which it was evaporated.

Mr. F. NEWMAN, in reference to the author's contention that Bazin's two coefficients (β, C) were not reliable, said he ought at the same time to have stated that the conditions under which the formulas were obtained, which he now brought before the Institution, were very different from those of Darcy and Bazin. The average width of the section of canal was 180 feet to 200 feet, and the depth in the center was only about 9 feet or 10 feet. The ratios of the depth to the width in the experiments made by the author were as one to twenty, while those of Darcy and Bazin were much less. In Fig. 3 was represented exactly the proportion that the depth bore to the width in the author's experiments. Attempts to reach the coefficient for velocity in a section in this must differ very much from one in the other on account of the greater excess of the wetted perimeter in the case of the canal. The section of the channel, as shown by the author, did not fairly represent the canal as it was actually constructed. That would represent a depth of 20 feet. With regard to the mean velocity the curve shown was Darcy's; if a horizontal line were drawn, which unfortunately he had not shown on the diagram, at one half the depth of the water, and a point were taken on the surface one-third of the half width from the border, and a verti-

Fig.3

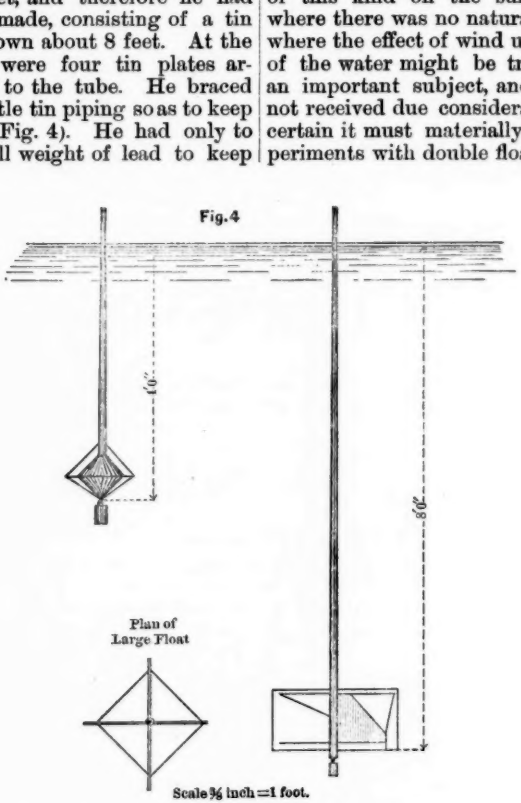


cal line drawn from that point, the intersection of those two lines would be a point of mean velocity. This nearly coincided with what was stated by Professor Unwin; in fact, the point 3.5 or 3.6 on the surface would represent the mean velocity as stated by him. The author also objected to certain formulas formed by Professor Moseley, and by Darcy and Bazin, and also to some experiments by Darcy on pipes; but those on the pipes had been remarkably correct, and they had been verified at Glasgow. Mr. J. M. Gale, M. Inst. C. E., gave the figures, the area of the pipe, and the rate of declivity, to the late Professor Rankine, who worked them out, and the slight difference between the coefficient of friction, as deduced by Gale on 4-feet pipes, and that given by Darcy amounted to less than one-thousandth per cent. That spoke well for the accuracy of Darcy and Bazin's experiments. Still, he thought that the author's remark to which he had referred should not have been made, because Bazin's coefficients were not applicable to a case of such enormous width in comparison with the area.

Mr. L. F. Vernon-Harcourt said that observers of discharge had in most cases adopted either the double float or the current-meter, and had generally considered that one or the other was exclusively the best. For instance, in Humphrey and Abbot's experiments on the Mississippi the double-float was adopted to the exclusion of the current-meter. Mr. Révy, on the contrary, held that the double-float was perfectly useless, and used current-meters alone for measuring the discharge of the Parana and La Plata rivers. Then, again, Mr. Gordon, in his experiments on the Irrawaddy, preferred the double-float, and the author had followed his example and rejected current-meters as possessing insurmountable difficulties in application, whereas Professor Unwin had carried out experiments satisfactorily with a current-meter. These facts pointed to the conclusion, which he had found in practice to be correct, that both instruments were suitable for measuring the discharge, and it was necessary to find out which was best adapted to the particular case. Of course, sometimes the double-float was the best, as, for instance, where the stream was sluggish and the channel fairly regular; but where the channel was irregular, where there were weeds—not merely floating along the stream, but growing in it—and where the bottom was uneven, the current-meter was far preferable. But there was another objection to the double-float, which the author had indeed mentioned, but passed over without the notice it deserved. namely, the effect of wind on the surface of water. According to the author, the primary effect of wind appeared to be the production of wave-motion, that it caused translation of the water only when long continued, and that, were it not so, every wind would produce a current on a lake or at sea. Of course, it was well known that long-continued winds did produce currents, as the trade winds created currents in the Atlantic Ocean. There were also currents on the Suez Canal, at one time in one direction, and at another time in another. These were produced by the prevalent winds in the Mediterranean and the Red Sea. Perhaps these would come under what the author would term winds of long duration; but if a strong wind was blowing up the Thames for even a short time it affected the tide. The author also said that the velocity at the surface of a stream was affected by the resistance of the air. If, then, a long-continued wind could produce a current, and, moreover, the resistance merely of the air affected the velocity of a stream, wind, even for a short time, must have some effect. This was brought to his notice in some float experiments he made in 1875 on the Firth of Clyde. In that case he had to measure how far the ebbing current would take down any floating material. He had a float made, which was especially designed to provide against the action of wind. The experiments were conducted at the best time of the year, in June and July, when there was no great amount of wind. The float was a tin tube above, and lower down it was brought out in shape of a cone, hollow inside. To make it float upright he had a weight attached underneath. It was about 4 feet long, and after a time he added some wings, with the object of offering a greater area to the tide where he thought that the wind would have no effect; but on a day when there was only a moderate amount of wind, blowing upstream, he found that instead of going down with the ebb tide,

it traveled in the opposite direction, though it reached about 4 feet below the surface (Fig. 4). At the same time an orange traveled up against the tide very much quicker. He concluded that the wind evidently did affect water at the depth of 4 feet, and therefore he had another float made, consisting of a tin tube, carried down about 8 feet. At the bottom there were four tin plates arranged radially to the tube. He braced these with a little tin piping so as to keep them secure (Fig. 4). He had only to add a very small weight of lead to keep it floating vertically in the stream. This float traveled down with the ebb when there was a strong wind blowing upstream, and when, of course, an orange would have been practically worthless. During the course of his experiments on the Firth of Clyde, he, several times, on calm days, started his two floats (Fig. 4) and an orange, as a surface-float, together, and observed that, whilst the calm continued, the three floats traveled fairly together, the surface-float generally going rather in advance, and the deep float falling somewhat behind. Directly, however, any opposing wind sprang up, the motion of the floats was reversed, the deep float taking the foremost place, and the orange coming last in order. He had often wished that he had the means and opportunity of making experiments of this kind on the surface of a lake where there was no natural current, and where the effect of wind upon the surface of the water might be tried. This was an important subject, and one that had not received due consideration. He was certain it must materially affect the experiments with double floats.

Fig. 4



The reason advanced by Professor Unwin for the surface-velocity being less than the velocity some distance below, was that when there was an irregular bottom, the water striking against the shoals was checked, and being forced upwards imparted its loss of velocity to the surface. He could not accept this, because it seemed to him that the lowest layers of water, being checked, must impart their loss of velocity to the lower portions first. He did not see any reason why the lowest stratum of water should arrest the motion at the surface

more than it did those layers nearer the bottom.

The author evidently thought that the only formulas that could be satisfactorily got from his experiments were those in which the mean velocity could be expressed in terms of maximum velocity; but, unfortunately, a formula of this kind, though very useful on rivers and canals for determining the discharge, would be quite useless in determining what the discharge would be after a channel had been enlarged or upon a new canal. Therefore, in such a case as that some kind of formula was required other than one which merely dealt with the mean velocity. He quite admitted that an experiment in an existing stream, from which the mean velocity was got, was much better than anything computed from a mere formula; but he would be very glad if the author could, by means of the valuable experiments he had made, deduce some formula that would give the discharge in terms of the slope, because that was the only way in which a formula could be useful for rivers that had been enlarged, or for new canals.

Mr. B. T. Moore said the author stated in his paper that the main object of all his hydraulic experiments was the determination of the discharge of large canals and rivers. The quantity of water which passed down a stream in a given time was equal in volume to the solid contained between a transverse section of the stream, a portion of the bed, a portion of the free surface, and a certain surface extending from the free surface to the bed and from bank to bank. This last surface was of a curved form, and generally convex at every point down the stream, whether it was supposed to be cut by horizontal or by vertical planes. The only practical way of measuring a quantity of that sort was by obtaining a sufficient number of parallel sections made by planes perpendicular to the transverse section, and equidistant from each other. When the areas of these sections, and their common distance were known, the prismoidal or other formulas of approximation would give the area to any required degree of exactness. This was the method he had always used when gauging rivers, and it was also the method which the author had adopted. But the areas of the parallel sections could only be found by means of velocity-measurement, and hence it was that the velocity-measurement was such an important matter. The author took a number of velocities at short distances from the surface towards the bottom, (generally about 1 foot) from which he plotted a curve, and found its area, making allowance for that portion of the curve which was below the lowest velocity and above the bed of the stream. But the author himself soon found that that was a very tedious and troublesome business; and, moreover, as it was of extreme importance that all velocities should be taken as nearly as possible at the same time, it became necessary to find some quicker means of doing the work. He was thus driven to seek some formula which would give the mean velocity at any vertical from two observations of velocity, one above and one below the half depth. Now, how did he arrive at that formula? He assumed that the form of the vertical velocity-curve was a portion of a parabola with its axis horizontal. If that assumption were made there was no necessity for a single practical experiment. By a simple mathematical transformation an equation could be obtained which would give the quantity sought. That equation was the following:

$$4(3\theta^2-3\theta+1)U=V_{\theta h}+3(2\theta-1)^2V_{h\frac{3\theta-2}{2\theta-1}} \quad . \quad (A)$$

in which h was the depth of water, and θ a fraction which might have any value not exceeding $\frac{1}{2}$; U was the mean velocity, and the two measured velocities were represented by the letter V, one being taken at the depth θh below the surface, and the other at the depth

$$\frac{h}{S}\cdot\frac{3\theta-2}{2\theta-1}.$$

If $\theta=0$, equation (A) became

$$4U=V_0+3V_{\frac{2}{3}h} \quad . \quad . \quad . \quad (B)$$

This was the principal formula mentioned by the author, and gave the mean velocity in terms of the surface velocity, and the velocity at two-thirds of the depth.

Again, if the quantity $3(2\theta-1)^2=1$, so as to make the coefficients of the two

measured velocities the same, the resulting equation was

$$2U = V_{0.211h} + V_{0.789h} \quad . \quad . \quad . \quad (C)$$

This was the last formula at which the author said he had arrived, his object in seeking it being to obtain a formula which should be applicable to a double submerged float, and should give the mean velocity from one observation. This last formula was not convenient for rapid work in the field, where the depth at which a velocity had to be taken has so often to be calculated mentally, and it was of no particular value for any other instrument except the double submerged float.

Another very convenient formula might be obtained from the general formula (A) by giving θ the value $\frac{1}{12}$. This formula was

$$37\ U = 12\ V_{\frac{1}{12}h} + 25\ V_{\frac{7}{10}h} \quad . \quad . \quad . \quad . \quad (D)$$

and an approximate form of it, good enough in many observations, was

$$3U = V_{\frac{1}{12}h} + 2V_{\frac{7}{10}h}.$$

The quantities $\frac{1}{12}h$ and $\frac{7}{10}h$ were easily calculated mentally, and they were the only quantities required to be known during the actual measurement of the velocities.

The formulas (B), (C) and (D) could all be used with current-meters as well as with floats, but the last was the most convenient, as it avoided surface velocities, and was very easy of application. But it was not every observer, or every experimenter, who accepted the theory that the velocity-curve was a parabola with its axis horizontal. On the contrary, some of the best experiments tended to show that the curve was better represented by a parabola, with its axis vertical, and its vertex in the bed, at least as far up as the maximum velocity. If that was assumed to be the case, it would result in an entirely different formula. It was quite possible that the two formulas might give the same, or nearly the same, mean velocity. Considering that there were two rival theories as to the form of the velocity-curve, and that they gave two totally different formulas, he thought great caution should be observed in adopting any formula depending on two measured velocities, and that it was safer to trust to a formula based on three velocities, which would always give the area of the velocity-curve to a close degree of approximation whatever the form of the curve might be.

Fig. 5 would render the subject clearer. The curve shown was a parabola with its axis horizontal and focus at F. Whatever the true form of the velocity-curve might be, it was always curved in one direction, and was generally much flatter than the curve shown. Now as the radius of curvature of a parabola varied from half the latus-rectum to infinity, and as the latus-rectum might have any value, a parabola could always be found which should pass through three points of the velocity-curve, and should coincide with it very closely throughout. The area of this parabola was easily calculated from three known velocities, and it could only differ very slightly from the area of the true velocity-curve. In Fig. 5 the three lines marked V_s, V_m, and V_h represented the three measured velocities, V_s being taken at the small distance δ below the surface, V_h at the same distance above the bed, and V_m at half the depth.

The area between the velocity-station and the curve was then given by the formula—

$$\text{Area} = \frac{h-2\delta}{6}\left\{ V_s + V_h + 4V_m \right\} + V_s\,\delta + V_h\,\delta \quad . \quad . \quad . \quad (1)$$

if the two exceedingly small triangles at a and c were neglected. But in practice it was seldom necessary to take into account the small distance δ. It was generally sufficiently accurate to consider V_s the velocity at the surface, and V_h the velocity at the bottom. The formula for the area thus becomes—

$$\text{Area} = \frac{h}{6}\left\{ V_s + V_h + 4V_m \right\} \quad . \quad . \quad . \quad (2)$$

The difference between (1) and (2) was

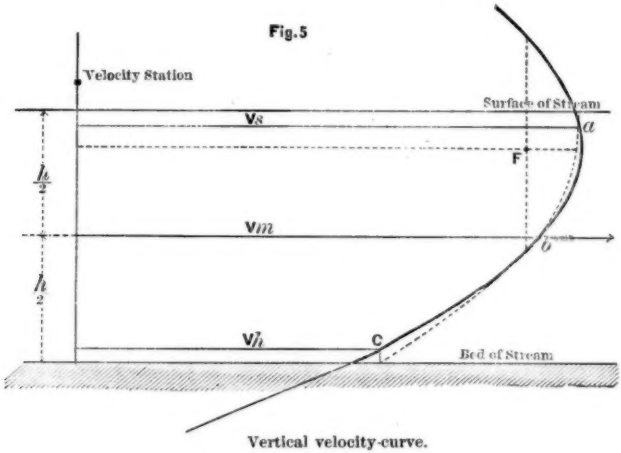
$$\frac{2}{3}\delta(V_s + V_h - 2V_m).$$

Now as δ was a small quantity, and as the velocity at mid-depth never differed

much from the arithmetic mean of the velocities at the surface and bottom, the error made by using formula (2) in place of formula (1) was always very small.*

The same conclusion would follow from an inspection of Fig. 5, in which the dotted line represented the curve resulting from the supposition that V_s and V_h were the surface and bottom velocities. It would be seen at once that the area bounded by the dotted curve was too small above the half depth and too large below it, and that on the whole it could only differ from the area in question very slightly. In general the velocity-curve was much flatter than the curve drawn, which was purposely much curved for the sake of giving distinctness to all parts of the figure.

Fig. 5



Vertical velocity-curve.

So far as he had spoken only of the use which was to be made of these three velocities and the parallel sections, but now he would consider how these velocities were to be obtained, for that was a most important thing. The author put his faith entirely in floats, not having a good word to say for any current-meter. It was not his intention to discuss all the objections that were raised to current-meters, but he would make this remark, that current-meters were indispensable and floats were not. Some years ago he had measured the discharge of the Nile. He had to do that at low Nile, which took place about the 21st of June. All the velocities had to be taken in three days, because the river remained pretty well constant during that time but no longer. Now, even at low Nile the best site which he could select was 1,600 feet or 1,700 in width, with a mean depth of over 10 feet. There were four sections extending to ¼ mile, nearly equi-distant from each other. The method he adopted was to measure the discharge through each of those sections, and compare them together as checks upon each other. There were sixteen principal velocity-stations in each section. That was the smallest number with which he could operate on such a large scale. That meant two hundred velocities, and all these two hundred velocities had to be taken within three days. Egypt, in June, was not a country in which one could work late in the day. All the work had to be done before ten o'clock in the morning, after which time the heat of the sun put a stop to field work. He would ask how the work could have been done by floats? As to taking velocities at depths where a number of observations had to be taken to get one velocity, it would have been

* In a velocity-curve taken on the Thames the following numbers were found:

V_s = 160.4 feet per minute.
V_m = 149.6 " "
V_h = 135.9 " "

and δ was 6 inches.

$\therefore$ Difference above $= \frac{2}{3} \times \frac{1}{2}(160.4 + 135.9 - 2 \times 149.6) =$ 1.27 square feet, the whole area being 244 square feet.

simply out of the question. That was an instance where current-meters were absolutely essential if work had to be done in a given time. The author had stated that floats were cheap, and current-meters expensive. But were floats cheap? If they were used, a number of skilled observers were necessary, and such observers were by no means cheap. The expense incurred in this way would soon pay for a considerable number of current-meters. One person could use three current-meters at the same time without the slightest difficulty. He could put one near the bottom and one near the surface, and a third in the middle, and get all the observations at the same time. A great saving of time was thereby secured, and the velocities were taken together, which was also an important point. He was speaking now of using the current-meter by means of suspension. Any contrivance for fixing the current-meters in position was a cause of great difficulty and delay. The author objected to current-meters in suspension, and spoke of the uncertainty of orientation. But Professor Unwin had pointed out that there was not much in that objection about orientation, and that if a current-meter was set at 20° from the direction of the current the error in its registration would not exceed 6 per cent. of the velocity, and if allowed to swing 20° on each side of the true direction the error would not be more than 3 per cent. Mr. Moore had used suspended current-meters very freely, but had never seen one swing 20° from the normal on each side; 10° would be an ample allowance to make. But with an allowance of 10° there was little to be said on account of this defect in orientation, because the error would not amount to more than $\frac{3}{4}$ per cent. The author also objected to current-meters being suspended, because there might be an error in position as regarded depth; but if the stream was a slow one, the suspending line was very nearly vertical, and there was no question of error in the position of the meter. If the stream was rapid, of course the line, even when the current-meter was loaded, would be drawn back a little from the vertical, but in that case there was a compensation. The variation of velocity from point to point vertically was very small compared with the velocity, when the velocity was considerable, and therefore it was not of so much importance whether the meter was a little above or below the level where it was intended to be. In his book the author showed a current-meter fixed to an apparatus attached to two pontoons; the current-meter was deprived of its tail, and was held in a position of restraint parallel to what the author supposed to be the axis of the stream; but if the current-meter was allowed to hang freely, it would find out the axis of the stream for itself, much better than could be done from the surface. The author raised several other objections to current-meters, which he could not accept, but he would not discuss them, because he had already taken up too much time. In his opinion the best way to obtain the mean velocity, and from it the area of a vertical velocity-curve was to use a current-meter which admitted of being lowered from the surface to the bottom with tolerably uniform velocity, and also of being drawn up again with uniform velocity. It was not necessary that the velocity going down should be the same as the velocity coming up. The whole reading, divided by the time, gave the mean velocity; nothing could be simpler. The mean velocity was got by one reading, and the elaborate contrivance of the electrical bell and apparatus was got rid of. He did not like those arrangements, because perhaps just at the very critical moment something might go wrong with the insulation, or the battery might not work, and so the current-meter would be rendered useless, or would give incorrect results.

Mr. E. A. Cowper wished to make one or two remarks in reference to the question raised in the paper on the instability of the floating gauges. Balls $1\frac{3}{4}$ inch in diameter, were spoken of, weighing but little more than the water displaced. They were kept from sinking by a thread and small float at the top. It seemed to him that it was not necessary to have such a small float. He would prefer to have two boards 1 foot square put across each other for the lower float, or at all events a large surface on which the water could act. But the point to which he wished to draw attention was this, that if the water on the surface went faster than the water below, it drew the string aside horizontally, and lifted the lower gauging board higher than it ought to be, as

pointed out in the paper. If, on the contrary, there were three threads hanging from three points at a distance apart on the top float, the lower float would always be at the proper depth from the level of the water, unless the current was so strong as to slack the threads altogether. It seemed to him to be a simple form of float, ensuring the exact depth of the lower float below the surface of the water. He wished to ask what price was charged for the water for irrigation? He understood that the experiments were all for the purpose of finding out what quantity of water to charge for, but the price was not stated, nor how the water was measured and sold to the natives.

Sir Frederick Bramwell said three points had been touched upon by the previous speakers, about which he wished to say a few words. One was as to the effect of wind on the surface of the water —whether it was merely skin deep, or was effective in moving the water along. Recently he was at Niagara, and when crossing below the Falls in the ferryboat, he asked the boatmen whether there was much variation in the height of the river. Their answer was that it depended not so much upon the season as on the direction of the wind. He crossed on several occasions, and one day the boatman pointed out to him that the water in the river below the Falls had risen 12 inches, entirely in consequence of a change of wind acting on the water above. That was good evidence that the direction of the wind either retarded or augmented the flow of the water. He therefore thought it must be regarded as effective far below the surface. Mr. Baldwin Latham objected to the possibility of there being a greater evaporation than was equal to the rainfall. It appeared to him, however, that all the rivers on the face of the earth must be derived from the excess of evaporation from the sea over the rainfall on the sea. The last speaker had suggested that if a current-meter had not got its tail, and was not allowed to follow the direction of the stream for itself, it would give an inaccurate result; but was that so? Was it not desirable that the current-meter should be fixed in a line in the axis of the stream, if it was required to know, not what was the maximum flow of water in some one direction at that particular point, but what was the flow of water in the direction of the river? It was quite clear that there were eddies all through the river. If there were not, the flowing river would deposit the suspended matters, just as they were deposited in a quiescent lake. What was wanted, therefore, was not that the current-meter should seek out for itself the point of strongest current, and should sway backwards and forwards as the current varied, but that the meter, always pointing in one direction, should give the velocity in that direction from time to time, because that when multiplied into the cross section would give the true velocity, and not the velocity that would be attained if the meter were at liberty to wander into the line of greatest current at the time being.

Mr. J. B. Redman directed attention to the well-known treatise by the late Major Rennell on ocean currents traced by him to the winds. Amongst others a local one at the back of the Goodwin was quoted, which increased in intensity after a continuance of S. W. gales. The recent abnormal addition to the tidal column of the Thames, an increase of 5 feet, or 25 per cent. as regarded altitude, due almost entirely to gales of wind, must necessarily have been accompanied by a corresponding increase of velocity.

Mr. Baldwin Latham said that the amount of evaporation from the sea was equal to the amount of rain falling on the sea and the volume of water carried by streams from the land into the sea. It was obvious that, taking equal areas, the depth of evaporation from the sea was less than the depth of the rainfall on the land, as the quantity of water carried by rivers and streams into the sea from the land was less than the rain which fell on the land. The excess of depth of rainfall on the land arose from the smaller area of land compared to that of sea, evaporation also taking place from the land as well as from the sea which vapor, on condensation, fell again on the land in the form of rain or dew.

Major Allen Cunningham observed, in reply to criticisms as to the suitability of the Ganges Canal for experiments on the flow of water, in consequence of its being laid out in reaches closed by obstructed falls, first, that irrigation canals in hot climates were of such enormous importance, both financially and in the interests

of humanity, where the very existence of immense populations sometimes depended on them in seasons of drought, that the conditions of the flow of water in them, even if peculiar, were of more public importance than those of flow of water in channels wholly natural. But the condition in question, of obstruction at the tail of each reach, was in no way peculiar; it was common to most fresh-water canals and also to most rivers of small size in highly civilized countries.

A more important objection had been raised by Professor Unwin as to the position of the two sites, at which most of the experiments were made, as being within the influence of the obstruction at the tail of the reach. The reach was $9\frac{1}{2}$ miles long, and the sites in question were $5\frac{1}{2}$ and $4\frac{3}{4}$ miles from the obstruction; the fall of the bed was 11.5, 6.1, and 4.9 feet in those lengths, whilst the raised crest of the falls at the tail of the reach was 5.1 feet high, or 0.2 feet above the bed of the lower site; but there were also means of raising the crest-level temporarily about 4 feet more, or over 4 feet above the bed of the lower site. This might, no doubt, be an objection to the position of both sites; but apart from that they were exceptionally favorable for experiment, from their situation in a straight 3 mile length of channel with regular masonry banks, and advantage which could not have been secured elsewhere. The power of varying the obstruction at the lower site was the source of, perhaps, the most important feature of the experiments, namely, the great range of conditions and of consequent results; for example, surface-slope 480 to 24 per million; velocity, 7.7 feet to 0.6 per second, &c., which were obtained at a single site, and in some cases without change of depth at the site. In no other large experiments had this great range been approached, nor could it be approached probably without artificial regulation; whilst without a wide range it was not likely that general laws of fluid motion would not be discovered, nor a single formula be properly tested.

It was suggested that surface-slope should, if possible, be deduced from the surface-fall in the same length, say 50 feet, as the run through which the floats were timed. The "slope-length" should certainly be the shortest practicable, but it would be impossible to measure the fall of the free surface in a 50 feet length in any ordinary stream. The oscillations of the free surface, often $\frac{1}{2}$ inch, and the awkward position of the body and eye, necessarily above that surface, precluded determining the minute quantity in question, say $\frac{1}{8}$ inch in 50 feet, with accuracy. The introduction, as proposed, of a baulk of timber 50 feet long into the running water for carrying micrometers would ruffle the surface. The only hopeful way of doing it appeared to be to lead the water from two fine nozzles 50 feet apart, similar, and similarly placed into two still-water boxes the difference of water-level in which could be accurately read with a differential hook-gauge or differential Pitot's tube; but no doubt the result would be affected by any want of similarity in the nozzles or in their position.

The objection to floats, that though they were cheap, their use was expensive from the frequent repetition, and therefore great length of time required to determine properly an "average velocity," was undoubtedly an important one. If it was true that an "average velocity" could be determined, as apparently suggested by Professor Unwin, in three minutes with a current-meter, then the current-meter should, if certain improvements could be effected in it, supersede the use of floats in suitable channels. The drawbacks to the current-meter were discussed at length in Chapter XXIII. of the Roorkee experiments. The objection that floats, even when deeply submerged, were seriously affected by wind, was contrary to most recorded experience. The instance given by Mr. Vernon Harcourt of a large sub-float with 4 feet immersion being driven by a high wind against tide was extraordinary; the ascribing this effect to the wind was contrary to all the Roorkee experience. Out of many hundred cases of upstream wind, not a single instance was noticed of even a surface-float (much less of a sunken float) being driven upstream by the wind; but the instance in question was in a tidal channel; in such a channel there were often counter sub-currents at certain states of the tide, so that this instance was not certain evidence of wind effect.

The double-float proposed by Mr.

Cowper had two great defects; the large triangular surface-float was much too large, and the three-cord connector exposed much more resistance to the current than a single cord of the same strengh would; these were the two worst faults in a double-float.

The statement that in the Mississippi experiments the double-float was so badly designed that its connector exposed one and a half time the area of the sub-float to the current depended probably on the description given in the original Mississippi report, in which the connector was said to have been $\frac{8}{10}$ inch thick; this has since been reported to be a misprint for $\frac{1}{10}$ inch, which reduced the ratio to 8:10.

Attention had been drawn by Mr. Baldwin Latham to some properties of transverse velocity curves in his own experiments, viz. (1) the maximum velocity being over the deepest channel; (2) the existence of two maxima on each side of a central shallow; (3) a depression in the curve near a shoulder in the bed; all these points were obvious in the Roorkee experiments. (Vol. I., pp. 257-260, and Pl. xxxv., xxxvi.) The statement of the maximum and mean velocity past a vertical being at definite depths, viz. at $\frac{1}{3}$ and at $\frac{2}{3}$ of the full depth did not agree with experiment; the maximum velocity might lie anywhere between the surface and mid-depth, and the mean velocity line varied in position with it, but the curve was so flat that the velocity at $\frac{6}{8}$ depth was, except near the banks, an approximation to the mean. The great variability of the coefficient (C) in the Chezy formula ($V = C \times 100 \sqrt{RS}$) was the very point towards which most of the Darcy Bazin and Roorkee experiments were directed.

The accuracy of the Darcy Bazin experiments on which so much stress had been laid had never been questioned. The suggestion of Mr. Newman that the failure of their coefficients when applied to the Roorkee results was due to the disparity of proportions of the Darcy Bazin canals and the Ganges canal was very likely correct, and amounted to an admission of the want of generality of those coefficients, as urged in the paper.

One main result of the Roorkee experiments was that approximation to mean velocity was more likely to be attained by direct velocity-measurement than by surface-slope measurement; but it could not be fairly said that scarcely any light was thrown on the use of formulas involving the latter. Much special experiment was done with this very aim, and with the definite result that Kutter's formula was the only one (not requiring velocity-measurement) of pretty general applicability, and would under favorable conditions give results differing by not more than $7\frac{1}{2}$ per cent. from actual velocity-measurements. This was surely a definite and important result.

The process used by Mr. Moore for discharge-computation was substantially that used in the Roorkee experiments. The velocity-formulas given by him were mostly those of, or easily deducible from that work (pp. 211–213, vol. I). The spacing at 0.211 and 0.789 of the full depth, Fig. 6, approximately at $\frac{2}{10}$ H and $\frac{8}{10}$ H, which were simple fractions, had great advantages. It gave the best general approximation possible, with only two ordinates for any curve whatever. The two velocities could be measured at one operation, which saved time, with a special double-float. When the two velocities were separately measured, the result was at once found as the arithmetic mean, which saves computation. Next, it was hardly fair to describe the taking the vertical curve to be a parabola with horizontal axis as a mere assumption. The curve was most likely not a parabola, but the amount of evidence that it did approximate very nearly to a parabola with horizontal axis, sufficiently for computing approximations, was now very great (chap. xi., vol. I). The parabola was by no means so accommodating a curve as represented. Thus, assuming a horizontal axis (Fig. 6), it could only be fitted to three points, whilst forty-five of the Roorkee curves had more than three, and ten of them have ten measured ordinates. This amounted to a severe test. On the other hand, the placing the axis vertical was certainly wrong, being incompatible with a maximum velocity-line below the surface, a condition the evidence of which was overwhelming.

The only evidence adduced by Mr. Latham as to the convexity or concavity of a river when rising or falling was very indirect; the observed facts might well be due to other causes; for instance,

the set of driftwood towards the banks might be due to wind or to surface-currents.

As to evaporation, dryness of the air was probably the most important exciting cause. In northern India the dryness of the air during the hot winds was excessive, the difference between dry and wet bulb thermometers being often 40°; so that evaporation must then be active. The smallness of the evaporation from the canal at Roorkee seemed to be due to the coldness of the water. The use of oil inside an evapometer would be hardly safe, as a little oil would probably escape and form a surface skin over the water, and so reduce the evaporation.

At a recent meeting of the Academie des Sciences, M. Bertrand reported that he had been present with M. Du Moncel at experiments which appeared to absolutely confirm the correctness of the law formulated by M. Marcel Deprez, viz.: (1) The intensity of an electric current remaining the same, whatever be the speed of the motor, the static effort does not change; and (2) in a machine worked by a current, the speed may be doubled, quadrupled, or decupled, without the intensity of the current varying. M. Du Moncel added that during the experiments the resistance of the circuit had been varied without changing the intensity of the current.

Fig. 6

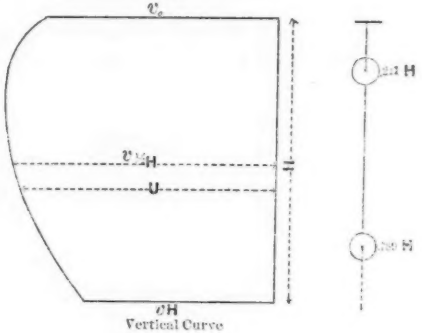
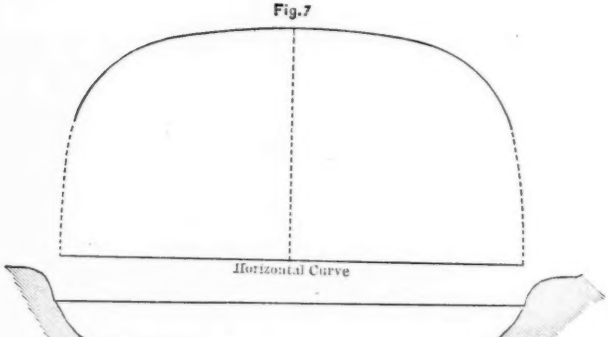


Fig. 7



CROSS SECTION OF CHANNEL.

UNDERGROUND TEMPERATURE.

From "Nature."

II.

E. We now Proceed to a Comparison of Results.

The localities at which definite results have been obtained may thus be classified:

1. Metallic mines. 2. Coal mines. 3. Wells and wet borings. 4. Tunnels.

1. The mines at Przibram in Bohemia, with a depth or 1900 feet, are in very quartzose rock, and give a very slow rate of increase, viz. 1° F. in 135 feet. As all the shafts are in lofty hills, an allowance of $\frac{1}{15}$ may be made for convexity, leaving 1° F. in 126 feet. Quartz is found by Prof. Herschel to have a conductivity of about .0086.

The mines at Schemnitz in Hungary, with a depth of 1368 feet, give an average rate of 1° F. in 74 feet, the rock being a green hornblende-andesite (in German, *Grunstein-Trachyt*), which is a compact fine-grained, crystalline, more or less vitreous rock. Prof. Lebour estimates its conductivity as being probably nearly the same as that of Calton Hill trap-rock, which Prof. Herschel found to be about .0029.

2. The principal results from coal mines are as follows:

The mines of the Société Cocqueril at Seraing (Belgium), with a depth of 1657 feet, give an average rate of 1° F. in 50 feet. The rock is coal shale. Prof. Herschel found for shale the low conductivity .0019.

The mines of Anzin, in the north of France, with a depth of 658 feet, gave in the deepest shaft an increase of 1° in 47 feet.

Rosebridge Colliery, near Wigan, with a depth of 2445 feet, gave a mean rate of 1° in 54 feet.

The four following are in the East Manchester coalfield.

Astley Pit, Dukinfield, with a depth of 2700 feet, gave a mean rate of 1° in 72 ft.

Ashton Moss Colliery, with a depth of 2790 feet, gave 1° in 77 feet.

Bredbury Colliery, with a depth of 1020 feet, gave 1° in 78.5 feet.

Nook Pit, with a depth of 1050 feet, gave 1° in 79 feet.

South Hetton Colliery, Durham, with a depth of 1929 feet, including a bore hole at bottom, gives very consistent observations at various depths, and an average rate of 1° in 57.5 feet.

Boldon Colliery, between Newcastle and Sunderland, with a depth of 1514 feet, and excellent conditions of observation, gives an average rate of 1° in 49 feet.

Kingswood Colliery, near Bristol, with a depth of 1769 feet, and remarkable consistency between observations at various points, gives 1° in 68 feet.

Prof. Phillips' observations in Monkwearmouth Colliery, published in *Phil. Mag.* for December 1834, showed a temperature of 71.2 in a hole bored in the floor of a recently exposed part at the depth of 1584 feet. The surface of the ground is 87 feet above high water, and the mean temperature of the air is assumed by Prof. Phillips to be 47.6. If, as usual, we add 1° to get the soil temperature, instead of assuming, as Prof. Phillips does, that the temperature 100 feet deep is identical with the air temperature at the surface, we obtain a rate of increase of 1° in 70 feet.

3. The following are the most trustworthy results from wells and borings:—

The Sperenberg bore, near Berlin, in rock salt, with a depth of 3492 English feet, to the deepest reliable observation, gave an average of 1° in 51.5 feet. This result is entitled to special weight, not only on account of the great depth, but also on account of the powerful means employed to exclude convection.

Rock salt, according to Prof. Herschel, has the very high conductivity .0113.

Three artesian wells in the chalk of the Paris Basin gave the following results:—

	Feet.	Rate, Feet.
St. Andre, depth of observation.	830	1 in 56.4
Grenelle.....	1312	1 in 56.9
Military School.....	568	1 in 56.2

An artesian well at St. Petersburgh, in the Lower Silurian strata, with a depth of 656 feet, gave about 1° in 44 feet.

A well sunk at Yakoutsk, in Siberia, to the depth of 540 feet, disclosed the fact that the ground was permanently frozen to this depth, and probably to the depth of 700 feet. The rate of increase of temperature was 1° in 52 feet.

Of the English wells in which observations have been taken the most important is that at Kentish Town, in which Mr. G. J. Symons, F.R.S., has taken observations to the depth of 1100 feet. The temperatures at different depths form a smooth series, and have been confirmed by observations repeated at long intervals. The only question that can arise as to the accuracy of the results is the possibility of their being affected by convection.

The well is 8 feet in diameter, with brickwork to the depth of 540 feet, and this part of it is traversed by an iron tube 8 inches in diameter, which is continued to the depth of more than 1300 feet from the surface. The tube is choked with mud to the depth of about 1080 feet, so that the deepest observations were taken under 20 feet of mud. The temperature at 1100 feet was 69°.9, and by combining this with the surface temperature of 49°.9 observed at the [Botanic Gardens, Regent's Park, we obtain a rate of 1° in 55 feet. These data would give at 250 feet feet a calculated temperature of 54.5, whereas the temperature actually observed at this depth was 56.1, or 1°.6 higher; the temperature of 300 feet and at 350 feet being also 56.1. This seems to indicate convection, but it can be accounted for by convection in the 8-foot well which surrounds the tube, and does not imply convection currents within the tube. Convection currents are much more easily formed in water columns of large diameter than in small ones, and the 20 feet of mud at the bottom give some security against convection at the deepest point of observation. It is important to remark that the increase from 1050 to 1100 feet is rather less than the average instead of being decidedly greater, as it would be if there were convection above, but not in the mud. The rate of 1° in 55 feet may therefore be adopted as correct.

The strata consist of tertiary strata, chalk (586 feet thick), upper greensand, and gault.

The Kentish Town temperature at the depth of 400 feet (58°) is confirmed by observations in Mr. Sich's well at Chiswick, which is 395 feet deep, and has a temperature varying from 58° to 57°.5.

The Bootle well, belonging to the Liverpool Waterworks, is 1302 feet deep, and the observations were taken in it during the sinking. The diameter of the bore is 24 inches, and convection might have been suspected but for the circumstance that there was a gradual upward flow of water from the bottom, which escaped from the upper part of the well by percolation to an underground reservoir near at hand. This would check the tendency to downflow of colder water from the top; and as the observations of temperature were always made at the bottom, they would thus be protected against convective disturbance.

The temperature at 226 feet was 52°, at 750 feet 56°, at 1302 feet 59°, giving by comparison of the first and last of these a mean rate of 1° in 154 feet. The circumstance that the boring ceased for six weeks at the depth of 1004 feet, and the temperature fell during this interval from 58°.1 to 57°.0, would seem to indicate an elevation of 1° due to the heat generated by the boring tool. An assumed surface temperature of 49° (only 0°.9 lower than that of the Botanic Gardens in London) would give by comparison with 57° at 1004 feet, a rate of 1° in 125½ feet, and by comparison with 59°, at 1302 feet, a rate of 1° in 130 feet, which last may be adopted as the best determination. The rock consists of the pebble beds of the Bunter or lower Trias, and the boring was executed at the rate of nearly 100 feet per month.

The boring at Swinderby, near Scarle (Lincoln), in search of coal, was carried to a depth of 2000 feet, with a diameter at the lower part of only 3⅛ inches—a circumstance favorable to accuracy, both as impeding convection and as promoting the rapid escape of the heat of boring. The temperature at the bottom was 79°, the water having been undisturbed for a month, and this by comparison with an assumed surface temperature of 50° gives a rate of 1° in 69 feet.

The rocks are Lower Lias, New Red Marl, (569 feet thick), New Red Sand-

stone (790 feet thick), Red Marl, and earthy Limestone.

The following results have been obtained from shallow borings. The first three were made under Sir William Thomson's direction, with a thermometer which could be read by estimation to hundredths of a degree:

Blythswood bore, near Glasgow, with a depth of 347 feet, gave a very regular increase of 1° in 50 feet.

Kirkland Neuk bore, in the immediate vicinity of the above, gave consistent observations at different seasons of the year from 180 feet to the bottom (354 feet), the rate being 1° in 53 feet. This bore passed through coal which had been "very much burned or charred."

South Balgray bore, near Glasgow, and north of the Clyde, with an available depth of 525 feet, gave by comparing the temperature at the bottom with that at 60 feet a rate of 1° in 41 feet.

Shale extends continuously from 390 to 450 feet from the surface, and the increase in these 60 feet of shale was 2°.02, which is at the rate of 1° in 30 feet. This rapid increase agrees with the fact that shale has very low conductivity, averaging .0019 in Prof. Herschel's experiments.

The only small bore remaining to be mentioned is that at Manegaon, in India, which had 310 feet available, and gave by comparing the temperature at this depth with that at 60 feet a rate of 1° in 68 feet. The rocks consist of fine softish sandstones and hard silty clays, the dip being 10°.

4. *Tunnels.*—The Mont Cenis Tunnel, which is about seven miles long, is at a depth of exactly a mile (5280 feet) beneath the creast of Mont Fréjus overhead. This was the warmest part of the tunnel, and had a temperature of 85°.1 F. The mean air temperature at the crest overhead was calculated by the engineer of tunnel, M. Giordano, by interpolating between the known temperature of the hill of S in Theodule and that of the city of Turin, the former being 430 meters higher, and the latter 2650 meters lower than the point in question. It is thus calculated — 2°.6 C. or 27°.3 F. If, according to our usual rule, we assume the ground to be 1° warmer than the air, we have 28°.3 to compare with 85°.1. This gives a rate of 1° in 93 feet; but, inasmuch as the convexity of the surface increases the distance between the isotherms, a correction will be necessary before we can fairly compare this result with rates under level ground. As a rough estimate we may take $\frac{7}{8}$ of 93, and adopt 1° in 79 feet, as the corrected result.

"The rocks on which the observations have been made are absolutely the same, geologically and otherwise, from the entrance to the tunnel on the Italian side for a distance of nearly 10,000 yards. They are not faulted to any extent, though highly inclined, contorted, and subjected to slight slips and slides. They consist, to a very large extent indeed, of silicates, chiefly of alumina, and the small quantity of lime they contain is a crystalline carbonate."

The St. Gothard Tunnel, which has a length of about nine miles, has been subjected to much more minute observation, a skilled geologist, Dr. Stapff having, under Government direction, devoted his whole time to investigating its geology and physics. He not only observed the temperature of the rock in the tunnel at very numerous points, but also determined, by observations of springs, the mean temperatures of the surface of the mountain at various points, and compared these with an empirical formula for air temperature deduced from the known mean temperatures of the air at Göschenen, Andermatt, Airolo, and the Hospice of St. Bernard. He infers from his comparisons a considerable excess of soil above air temperature, increasing from 2° C. at the ends of the tunnel to 6° C. at the crest of the mountain over the center of the tunnel. The highest temperature of the rocks in the tunnel was at this central part, and was above 30°.6 C. or 87° F. The soil temperature at the crest above it was about—0°.6 C., or 31° F., giving a difference of 56° F. The height of the crest above sea level was about 2850 m., and that of the tunnel at this part 1150 m., giving a difference of 1700 m. or 5578 feet. The rate of increase here is, therefore, about 1° F. in 100 feet; and if we apply the same correction for convexity as in the case of the Mont Cenis Tunnel, this will be reduced to about 1° F. in 87 feet, as the equivalent rate under a level surface. From combining his observation in all parts of the tunnel through the medium of empirical formulæ, Dr. Stapff deduces an average

rate of 1° F. for every 88 feet measured from the surface directly overhead. Where the surface is a steep ridge, the increase was less rapid than this average; where the surface was a valley or plain, the increase was more rapid. As this average merely applies to the actual temperatures the application of a correction for the general convexity of the surface would give a more rapid rate. If we bring the isotherms nearer by one part in 15, which seems a fair assumption, we shall obtain a rate of 1° F. in 82 feet.

Collecting together all the results which appear reliable, and arranging them mainly in the order of their rates of increase, but also with some reference to locality, we have the following list:

		Depth feet.	Feet for 1° F.
Boodle waterworks (Liverpool)		1392	130
Przibram mines (Bohemia)		1900	126
St. Gothard tunnel		5578	82
Mont Cenis tunnel		5280	79
Talargoch lead mine (Flint)		1041	80
Nook Pit Colliery	East Manchester coalfield	1050	79
Bredbury "		1020	78½
Ashton Moss "		2790	77
Denton "		1317	77
Astley Pit, Dukinfield		2700	72
Schemnitz mines (Hungary)		1368	74
Scarle boring (Lincoln)		2000	69
Manegaon boring (India)		310	68
Pontypridd colliery (S. Wales)		855	76
Kingswood colliery (Bristol)		1769	68
Radstock " (Bath)		620	62
Paris artesian well, Grenelle		1312	57
" " St. Andre		830	56
" " Military Sch'l		568	56
London " Kentish Town		1100	55
Rosebridge colliery (Wigan)		2445	54
Yakoutsk, frozen ground (Siberia)		540	52
Sperenberg, boring in salt (Berlin)		3492	51½
Seraing collieries (Belgium)		1657	50
Monkwearm'th collieries (Durham)		1584	70
South Hetton " "		1929	57½
Boldon " "		1514	49
Whitehaven " Cumberl'd.		1250	45
Kirkland Neuk bore (Glasgow)		354	53
Blythswood " "		347	50
South Balgray " "		525	41
Anzin collieries (North of France)		658	47
St. Petersburg, well (Russia)		656	44
Carrickfergus, shaft of salt mine (Ireland)		770	43
Carrickfergus, shaft of salt mine (Ireland)		570	40
Slitt mine, Weardale (Northumberland)		660	34

The depth stated is in each case that of the deepest observation that has been utilized.

F. In Deducing a Mean from these very Various Results, it is better to operate not upon the number of feet per degree, but upon its reciprocal—the increase of temperature per foot. Assigning to the results in the foregoing list weights proportional to the depths, the mean increase of temperature per foot is found to be .01563, or about $\frac{1}{64}$ of a degree per foot—that is, 1° F. in 64 feet.

It would be more just to assign greater weight to those single results which represent a large district or an extensive group of mines, especially where the data are known to be very accurate. Doubling the weights above assigned to Przibram, St. Gothard, Mont Cenis, Schemnitz, Kentish Town, Rosebridge, and Seraing, and quadrupling that assigned to Sperenberg, no material difference is made in the result. The mean still comes out 1° F. in 64 feet, or more exactly .01565 of a degree per foot.

This is a slower rate than has been generally assumed, but it has been fairly deduced from the evidence contained in the Committee's Reports; and there is no reason to throw doubt on the results in the upper portion of the above list more than on those in its lower portion. Any error that can reasonably be attributed to the data used in the calculations for the St. Gothard Tunnel and for the numerous deep mines of the East Manchester coalfield, will have only a trifling effect on the rates of increase assigned to these localities.

To obtain an approximation to the rate at which heat escapes annually from the earth, we will first reduce the above rate of increase .01566 to Centigrade degrees per centimeter of depth. For this purpose we must multiply by .0182, giving .000285.

To calculate the rate of escape of heat, this must be multiplied by the conductivity.

The most certain determinations yet made of the conductivity of a portion of the earth's substance are those deduced by Sir William Thomson by an indirect method, involving observations of underground thermometers at three stations at Edinburgh, combined with laboratory measurement at the specific heats and densities of the rocks in which the thermometers were planted. The specific heats were determined by Regnault, and

the densities by Forbes. Specific heats and densities can be determined with great accuracy in the laboratory, but the direct determination of conductivity in the laboratory is exceedingly difficult, it being almost impossible to avoid sources of error which make the conductivity appear less than it really is.

Prof. Herschel, in conjunction with a Committee of the British Association, has made a very extensive and valuable series of direct measurements of the conductivities of a great variety of rocks, and has given additional certainty to his results by selecting as two of the subjects of his experiments the Calton Hill Trap and Craigleith sandstone, to which Sir William Thomson's determinations apply.

From combining Prof. Herschel's determinations with those of Sir William Thomson, .0058 is adopted as the mean conductivity of the outer crust of the earth, which, being multiplied by the mean rate of increase, .000285, gives

$$16330 \times 10_{-10}$$

as the flow of heat in a second across a square centimeter. Multiplying by the number of seconds in a year, which is approximately $31\frac{1}{2}$ millions, we have

$$1633 \times 315 \times 10_{.4} = 41.4.$$

This, then, is our estimate of the average number of gramme degrees of heat that escape annually through each square centimeter of a horizontal section of the earth's substance.

AMERICAN PRACTICE IN WARMING BUILDINGS BY STEAM.

By the Late ROBERT BRIGGS, M. Inst. C. E.

From Proceedings of the Institution of Civil Engineers.

I.

THE application of steam to the warming of buildings in the United States originated with the late Mr. Joseph Nason, of Boston and New York, who died six or seven years ago. He was not only the first to make the attempt, but was also the originator, improver, and adapter of much that is essential, and now implicitly followed, in the general arrangement and details of the apparatus employed. He enjoyed the advantage of having been for a short time a pupil of Jacob Perkins in London, about 1840; and his earliest endeavor in America was to adapt the Perkins system of hot water inside small tubes for meeting the severity of the climate of America. The large extent of warming surface and the great strength presented by steam apparatus constructed of small and comparatively inexpensive wrought iron tubes, and the facility thereby afforded for transmitting heat in any direction from a central source, are merits which led to so rapid a development of this system of warming, that by 1860, or in less than twenty years, there were already many hundred establishments throughout America for the manufacture of the apparatus. With the maturing of the system are associated the names of Mr. J. J. Walworth, of Boston, brother-in-law and partner of Mr. Nason; Mr. Gregg, of New York; Mr. J. O. Morse, of New York, by whom, amongst other improvements, was introduced the method of closed circulation for working with steam below atmospheric pressure; Professor Mapes, of New York, who supplied a reliable steam trap; Mr. Miles Greenwood, of Cincinnati, to whom the author believes the arrangement is due of a coil or nest of tubes connected by return-bends; and Mr. Thomas T. Tasker, of Philadelphia, who introduced the first closed apparatus. For himself, the author may claim to have established certain characteristic shapes and dimensions now universally adopted for the "globe" stop-valves and for the fittings or couplings of the tubes.

Wrought Iron Welded Tubes.—In the construction of the apparatus for warming by steam, the prevailing practice in America is to employ wrought iron welded tubes, not only for the

mains, but also to a large extent for the radiating surfaces that diffuse the heat. The separate lengths of tubes are connected by wrought iron couplings, when in the same straight line; and when not so, by cast iron elbows, tees, branch-pieces, and return-bends. The so called coils or radiators usually employed for diffusing the heat are compact nests of tubes, sometimes arranged vertically by having their bottom ends screwed into a cast-iron box, and at other times placed horizontally and connected together by branch-tees and return-bends.

In the use of these tubes, the essential feature of practical importance is the employment of tapering screw-threads, externally upon the tube-ends and internally within the sockets of the couplings or fittings, as the means of securing durable steam-tight joints which can be readily made or unmade. The system of taper screw-threads for the tube connections is believed by the author to have been originated by Mr. Nason, with whom he became associated in 1846; at that time the threads were cut in a lathe.

Where any two metallic surfaces are to form a steam-tight joint, they must be brought together into complete contact, either with each other or with some elastic or yielding packing interposed between them; and the force compressing them together must be sufficient both to obliterate any inequalities in the surfaces themselves, and also to withstand any internal pressure tending to part them. In the case of the machine-finished surfaces of the external and internal taper screw-threads forming the tube joints, the required contact is no doubt mainly effected by the yielding of the metal of the tube under compression and of the socket under tension; while the thread itself serves to maintain the joint against the internal disruptive steam-pressure.

A screw-thread, either external or internal, cut or tapped by means of the usual workshop appliances, is far from possessing absolute accuracy in radius, either for the top of the thread or for the bottom of the groove; and presents also the defect known as "drunkenness," or want of uniformity in its inclination or pitch. In making a steam-tight joint, these imperfections have to be overcome by the application of force. The thread has also defects in angular form, arising from imperfections in the dies or taps, and from the metal having been torn instead of being cut clean in the formation of the thread. These defects are remedied by the use of a paste, made of white lead ground in linseed oil and mixed with about an equal quantity of dry red-lead. This material acts both as lubricant and as packing in making the joint; it fills all interstices in screwing the joint up, and sets hard on the first application of steam heat, or within a short time if left cold.

The leverage of the pipe-tongs or wrench, employed to screw the tube up, is largely multiplied by the action of the screw itself in converting the force into the longitudinal direction; and the tube end being moreover shaped as a conical plug of slight taper, the longitudinal force results in a greatly intensified pressure of contact between the threads of the tube and its socket. For example:—the ordinary pitch is $11\frac{1}{2}$ threads per inch for a tube of 1 inch nominal diameter inside or 1.31 inch actual diameter outside, and the taper of both the tube end and the inside of the socket is 1 in 32 to their axis. After the tube has been entered into the socket as far as it can readily be screwed up by hand, one quarter turn more with the pipe-tongs is generally sufficient to make the joint steam-tight. In this quarter turn of the tube the difference of diameter is in the ratio of $1 : 11\frac{1}{2} \times 16 \times 4$, and is therefore equal to 1-736th of an inch. Meanwhile the hand, acting on the tongs at a radius of usually about 16 inches, will move through an arc of 25 inches, giving thus an effective leverage of say 18,000 times. The bursting pressure thereby produced within the socket, or the compression upon the tube end, is concentrated on about $\frac{1}{2}$ inch length of the socket or tube, and the mean diameter is there about $1\frac{1}{4}$ inch; whence the pressure per square inch tending to burst the socket is roughly about 11,000 times the manual force exerted on the tongs, if the frictional resistance to turning be neglected.

In practice it is really by the frictional resistance to turning that the limit of bursting strain actually thrown upon the cast-iron socket in screwing up is determined. For supposing the limit be taken

to be a radial pressure of say 5,000 lbs. per square inch, it is seen from the foregoing that, after overcoming the frictional resistance to turning, an insignificant ½lb. would be all the extra force required on the pipe-tongs for throwing this great bursting strain upon the socket. Allow now 15 per cent. as the coefficient of friction—a value not unreasonable when it is borne in mind that points of metal are brought into contact, and that the great pressure between them destroys the continuity of the imperfect lubricant. Then neglecting the screw-thread, and regarding the tube end and the casting as a plain conical plug and corresponding socket, the resistance to turning, offered by the ½-inch length of tube-end of 1¼ inch mean diameter, is $5{,}000 \times 0.15 \times 3.1416 \times 1\frac{1}{4} \times \frac{1}{2} = 1{,}400$ lbs. acting at a radius of ⅝ inch, which is equivalent to about 55 lbs. exerted on the handle of the tongs of 16 inches radius. A skilled workman judiciously keeps well within this limit in the force which he exerts; and the cast-iron fittings being made according to the proportions hereafter given, the thickness of metal left outside the thread after tapping them is amply sufficient to stand the tensile strain so exerted, which rises probably to 10,000 or 12,000 lbs., say 5 tons per square inch of section of metal. In practice, moreover, it is found that a joint can safely be screwed up with the tongs through even one whole turn, instead of only a quarter turn; the wrought-iron tube-end yields permanently under the compression, while the limit of elasticity of the cast-iron socket is not exceeded, and hence, neither metal has its strength impaired. Complete facility is thus afforded for leading off bends or branches in any desired angular position.

The taper employed for the conical tube-ends is uniform with all makers of tubes or fittings—namely an inclination of 1 in 32 to the axis. Custom has established also a particular length of screwed end for each different diameter of tube. Tubes of the several diameters are kept in stock by manufacturers and merchants, and form the basis of a regular trade in the apparatus for warming by steam. A knowledge of all these particulars is, therefore, essential for designing apparatus for the purpose. The ruling dimension in wrought-iron tube work is the external diameter of certain nominal sizes, which are designated roughly according to their internal diameter. These nominal sizes were mainly established in the English tube trade between 1820 and 1840, and certain pitches of screw-thread were then adopted for them, the coarseness of the pitch varying roughly with the diameter, but in an arbitrary way utterly devoid of regularity. The length of the screwed portion on the tube end varies with the external diameter of the tube according to an arbitrary rule of thumb; whence results, for each size of tube, a certain minimum thickness of metal at the outer extremity of the tapering screwed tube-end. It is the determination of this minimum thickness of metal for the tapering screwed end of a wrought-iron tube which constitutes the question of mechanical interest.

The thread employed has an angle of 60°; it is slightly rounded off both at the top and at the bottom, so that the height or depth of the thread, instead of being exactly equal to the pitch, is only four-fifths of the pitch, or equal to $0.8 \times \frac{1}{n}$ if n be the number of threads per inch. For the length of tube end throughout which the screw-thread continues perfect, the empirical formula used is $(0.8\ D + 4.8) \times \frac{1}{n}$, where D is the actual external diameter of the tube throughout its parallel length, and is expressed in inches. Further back, beyond the perfect threads, come two having the same taper at the bottom, but imperfect at the top. The remaining imperfect portion of the screw-thread, furthest back from the extremity of the tube, is not essential in any way to this system of joint; and its imperfection is simply incidental to the process of cutting the thread at a single operation. From the foregoing it follows that, at the very extremity of the tube, the diameter at the bottom of the thread $= D - \left(\frac{2 \times (0.8D + 4.8)}{32n} + \frac{3 \times 0.8}{n}\right) = D - (0.05D + 1.9) \times \frac{1}{n}$. The thickness of iron below the bottom of the thread, at the tube extremity, is empirically taken to be $= 0.0175\ D + 0.025$. Hence the actual internal diameter d of any tube is found to be, in inches,

$d = D - (0.05D + 1.9) \times \frac{1}{n} - 2 \times (0.0175\ D + 0.025)$, or $d = 0.965\ D - 0.05\frac{D}{n} - \frac{1.9}{n} - 0.05$.

For the various sizes of tubes, ranging from $\frac{1}{8}$ inch to 10 inches nominal internal diameter, with their corresponding numbers of screw-threads per inch, the actual internal diameter d is expressed by the following Table I. in terms of the actual external diameter D.

TABLE I.—DIAMETERS OF WROUGHT-IRON WELDED TUBES FOR WARMING BY STEAM.

Nominal internal diameter of tube.	No. of screw threads per inch.	Actual internal diameter d in terms of actual external diameter D.
Inches.	No.	Inches.
$\frac{1}{8}$	27	$d = 0.9631\ D - 0.1204$
$\frac{1}{4}$ and $\frac{3}{8}$	18	$d = 0.9622\ D - 0.1556$
$\frac{1}{2}$ and $\frac{3}{4}$	14	$d = 0\ 9614\ D - 0.1857$
1, $1\frac{1}{4}$, $1\frac{1}{2}$ & 2	$11\frac{1}{2}$	$d = 0.9607\ D - 0.2152$
$2\frac{1}{2}$ to 10	8	$d = 0.9587\ D - 0.2875$

The figures derived from this statement which are of importance for practical use are presented in detail in the accompanying Table II. in a convenient order for reference.

The number of screw-threads per inch for the several sizes of tubes is here accepted from customary usage. It is the workman's approximation to the pitch practically desirable, and much reluctance must consequently be felt in calling it in question. Still it would have been better to investigate the general case upon the basis of a pitch ranging in closer accordance with the range of tube diameter. Thus the nominal $\frac{1}{2}$-inch tubes might have had 16 threads per inch; $\frac{3}{4}$ inch, 14 threads; 1 and $1\frac{1}{4}$ inch, 12 threads; $1\frac{1}{2}$ and 2 inches, 11 threads; $2\frac{1}{2}$ to $3\frac{1}{2}$ inches, 10 threads; 4 to 6 inches, 8 threads; 7 to 9 inches, 7 threads; and 10 inches, not more than 6 threads per inch. The existing numbers of threads, however, as given in Tables I. and II., are now too well established to be disturbed: at all events they must be taken in any statement of present practice.

The smaller sizes of tubes, up to and including $1\frac{1}{4}$ inch nominal inside diameter, are butt-welded, and are proved by hydraulic pressure to 300 lbs. per square inch. The larger sizes, commencing with $1\frac{1}{2}$ inch, are lap welded, and are proved to 500 lbs. per square inch by hydraulic pressure. The question of butt-welding or lap-welding is simply one of economy in manufacture: it is not easy to make lap-welded tubes of less than $1\frac{1}{2}$ inch inside diameter, while it is cheaper to make lap welded than butt-welded tubes of that size and upwards. The proving pressures here given are far below the ultimate strength of sound welded tubes; and when required the test is increased to double or treble these pressures.

Couplings.—The internal threads in the sockets or fittings, by which the tubes are connected together, require the same accuracy of workmanship as the external threads on the tube ends. In practice it is found that the straight couplings or sockets, connecting tubes in the same straight line, may be made of wrought iron, and may be tapped parallel instead of taper; they are sized to screw freely upon the small extremity of the tapering tube-end up to as far as half the length of the perfect thread on the tube; and they then become stretched by the force exerted in screwing them upon the remaining half length of larger diameter, and form their own taper to fit the tube. To allow for clearance between the ends of the two tubes united by the coupling socket, the length of the coupling is made as nearly $2\frac{1}{2}$ times the length of the perfect threaded tube-end as will suit the width of iron from which the coupling is manufactured. The minimum thicknesses of metal for the several sizes of couplings previous to tapping are given in Table III., with their corresponding diameters and lengths.

This parallel tapping is not altogether satisfactory for the straight couplings for tubes above $2\frac{1}{2}$ or 3 ins. nominal diameter, and the joints so made are not reliable if screwed up with the ordinary tongs; for elbows or tees the method is very unsatisfactory. Tapering threads are usual in the wrought-iron sockets for the "oil tubing" in petroleum wells, and for the "line pipe" through which the petroleum is conveyed; these tubes range from 5 to 10 inches in diameter, and are subjected to pressures of 1,200 to 1,600 lbs. per square inch; the tubes themselves are frequently burst by the pressures ex-

TABLE II.—STANDARD DIMENSIONS OF WROUGHT-IRON WELDED TUBES FOR WARMING BY STEAM.

Diameter of tube.			Thickness of metal.	Circ'mfere'ce.		Length of tube per sq. foot of		Transverse area.		Length of tube to contain one cubic foot.	Weight per foot of length.	Screwed ends.	
Nominal inside.	Actual inside.	Actual outside.		Inside diameter.	Outside diameter.	Inside surface.	Outside surface.	Inside diameter.	Outside diameter.			No. of threads per inch.	Length of perfect screw.
Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Feet.	Feet.	Sq. ins.	Sq. ins	Feet.	Lbs.	No.	Inch.
⅛	0.270	0.405	0.068	0.848	1.272	14.150	9.440	0.0572	0.129	2500.00	0.243	27	0.19
¼	0.364	0.540	0.088	1.144	1.696	10.500	7.075	0.1041	0.229	1385.00	0.422	18	0 29
⅜	0.494	0.675	0.091	1.552	2.121	7.670	5.657	0.1916	0.358	751.50	0.561	18	0 30
½	0.623	0.840	0 109	1.957	2.652	6 130	4.502	0.3048	0.554	472.40	0.845	14	0.39
¾	0.824	1.050	0.113	2 589	3.299	4.635	3.637	0.5333	0.866	270.00	1.126	14	0.40
1	1.048	1.315	0.134	3.292	4.134	3.679	2.903	0.8627	1.357	166.90	1.670	11½	0.51
1¼	1.380	1.660	0.140	4.335	5.215	2.768	2.301	1.496	2.164	96.25	2.258	11½	0.54
1½	1.610	1 900	0.145	5.061	5.969	2.371	2.010	2.038	2.835	70.65	2.694	11½	0.55
2	2 067	2.375	0.154	6.494	7.461	1.848	1.611	3.355	4.430	42.36	3.667	11½	0.58
2½	2.468	2 875	0.204	7.754	9.032	1 547	1.328	4.783	6.491	30.11	5.773	8	0.89
3	3.067	3.500	0.217	9.636	10.996	1.245	1.091	7.388	9.621	19.49	7.547	8	0.95
3½	3.548	4.000	0.226	11 146	12.566	1.077	0.955	9.887	12.566	14.56	9.055	8	1.00
4	4.026	4.500	0.237	12.648	14.137	0.949	0.849	12.730	15.904	11.31	10 728	8	1.05
4½	4.508	5.000	0.246	14.153	15.708	0.848	0.765	15.939	19.635	9.63	12.492	8	1.10
5	5.045	5.563	0.259	15.849	17 475	0.757	0.629	19.990	24.299	7.20	14.564	8	1.16
6	6.065	6.625	0.280	19.054	20.813	0.630	0.577	28.889	34.471	4.98	18.767	8	1.26
7	7.023	7.625	0.301	22.063	23.954	0.544	0.505	38.737	45.663	3.72	23.410	8	1.36
8	7.982	8.625	0 322	25.076	27.096	0.478	0.444	50.039	58.426	2.88	28.348	8	1.46
9	9.000	9.688	0.344	28.277	30.433	0.425	0.394	63.633	73.715	2.26	34.077	8	1.57
10	10.019	10.750	0.366	31 475	33.772	0.381	0.355	78.838	90.762	1.80	40.641	8	1.68

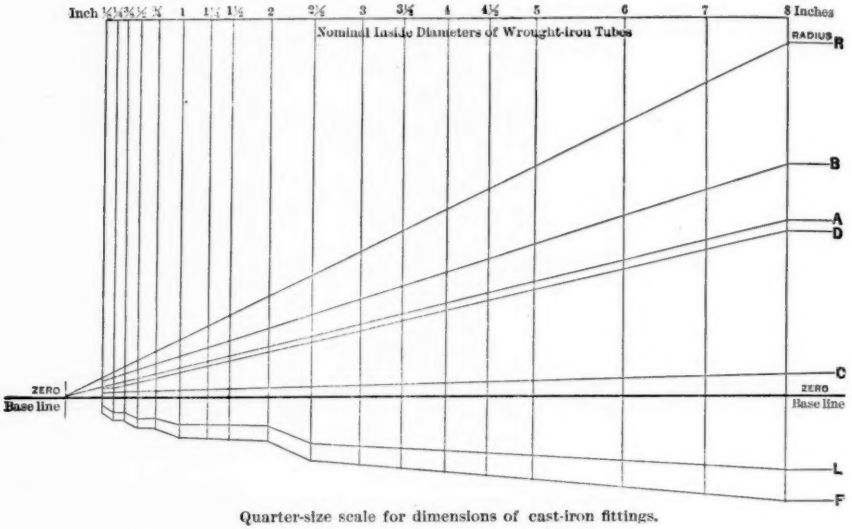
Taper of conical tube-ends, 1 in 32 to axis of tube.

In tubes for heating water or steam, the surface in contact with the products of combustion is to be taken as the effective heating surface, whether it be the inside or the outside of the tubes. But in tubes for heating liquids by steam, for superheating steam, or for transferring heat from one liquid or gas to another, the mean between the inside and outside surfaces is to be taken as the effective surface. In warming by steam, the outside surface exposed to the air is to be taken.

ceeding these, but the joints are perfectly reliable and unyielding. In the original construction of taper joints by Mr. Joseph Nason, the sockets of all sizes were made taper; the subsequent departure from this practice was a matter of cheapness in manufacture, and has never been a mechanical success.

The great power required for tapping a tapering thread in wrought iron, and the extra cost of making wrought-iron elbows and tees &c., have led to the employment of cast iron as the material for the fittings or couplings. Careful study having been bestowed upon the reduction of the material to the smallest quantity requisite, and upon the production of the cored castings with the least amount of preparatory work, these cast-iron fittings have now become universal for all joints of wrought-iron tubes, excepting only the straight couplings already mentioned. The scale drawn one quarter full size in Fig. 1 gives the exact dimensions for all parts of every possible elbow, tee, cross, or branch, for tubes of ¼ inch up to 8 inches nominal inside diameter. With these dimensions the least quantity of material, consistent with uniformity of strength, is here employed, and is arranged in the most compact form. A longitudinal section is shown in Fig. 2 of a 1¼ inch elbow, that is, the elbow suitable for a nominal 1¼ inch tube. The dimensions for this section are obtained from the scale by measuring, along the vertical line headed 1¼ inch, the height of ordinate intercepted between the horizontal base or zero line and the slant line marked by the letter corresponding with that on the section. In the two orifices, which have to be tapped subsequently for screwing upon the tube-ends, the diameter (twice the dimension marked D in the section Fig. 2) is cored just so much smaller than the smallest diameter

Fig. 1



Quarter-size scale for dimensions of cast-iron fittings.

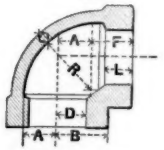
TABLE III.—DIMENSIONS OF STRAIGHT SOCKETS FOR COUPLING WROUGHT-IRON TUBES.

Diameter of tube.		Diameter of socket before tapping.		Thickness of socket before tapping.		Length of socket.
Nominal inside.	Actual outside.	Outside.	Inside.	Actual.	Birmingham wire gauge.	
Inches.	Inches.	Inches.	Inches.	Inch.	No.	Inches.
1/8	0.405	0.551	0.333	0.109	12	*7/8
1/4	0.540	0 699	0.431	0.134	10	1
3/8	0.675	0.834	0.566	0.134	10	1
1/2	0.840	1.011	0.699	0.156	8 1/2	1 5/16
3/4	1.050	1.239	0.909	0.165	8	1 1/16
1	1.315	1.527	1.143	0.192	6 1/2	1 1/4
1 1/4	1.660	1.893	1.487	0.203	6	1 1/4
1 1/2	1.900	2.166	1.726	0 220	5	*2
2	2.375	2 677	2.201	0.238	4	*2 1/4
2 1/2	2.875	3.190	2.622	0.284	2	2 1/4
3	3.500	3.870	3.245	0.313	—	3
3 1/2	4.000	4.404	3.724	0.340	0	3 1/8
4	4.500	4.992	4.242	0.375	00	3 1/4
4 1/2	5.000	5.491	4.741	0.375	00	3 3/8
5	5.563	6.186	5 311	0 438	—	3 1/2
6	6.625	7.236	6.361	0.438	—	3 3/4
7	7.625	8.358	7.358	0.500	—	4
8	8.625	9.489	8.364	0.563	—	4 1/4
9	9.688	10.539	9.414	0.563	—	4 1/2
10	10.750	11.720	10.470	0.625	—	5

* The three lengths marked by an asterisk are increased disproportionately for appearance.

of the root of the thread on the tube-end as will allow for removing the scale of hard iron from the casting by means of a drill; after which a taper tap, corresponding with the external thread on the tube-end, produces the internal thread in the socket, cutting with the proper taper a perfect thread inside the orifice through-

Fig.2



Longitudinal section of cast-iron elbow for 1¼ inch tubes (before tapping) quarter full size.

out the whole of its length, marked L in the section. There are, therefore, excepting in special cases, no imperfect threads in the cast iron fittings or sockets.

It may be well to explain, in regard to the construction of the scale shown in Fig. 1, that the ordinates, though here headed for convenience with the nominal inside diameters of the tubes, are spaced from the zero point at horizontal distances (half full size) equal to the actual outside diameters as given in the foregoing Table II. Of the slant lines, only two pass through the origin or zero point: namely, the line for the internal radius R of the back of the elbow, and the line giving the dimension A which is exactly half the radius R. Of the remaining lines, those giving the three dimensions B, D, C, intersect the base line a little to the left of the origin or zero point: which means that each of the three dimensions given by these lines contains some small increment in excess of the dimension that would be strictly proportional. For instance, with regard to the thickness C at the back of the elbow, it is evident that, if the metal in any casting, whether small or large, thin or thick, were truly homogeneous throughout both its skin and its interior portion, this thickness should then be strictly proportional to the diameter or radius of the bore; but for small castings a certain allowance of extra thickness is needed, in order to ensure soundness, as well as to compensate for the increased brittleness of thin cast iron. There will however be some size of fitting for which the normal strength of metal may safely be assumed, and for which therefore no extra thickness will be needed. On this assumption the thickness for the largest size is set out at the end of the scale furthest from the origin; while nearest to the origin is set out the thickness proper for the smallest size, in which the increment is at its maximum; and the slanting straight line drawn between these two extremes gives for intermediate sizes an increment which varies inversely as the diameter or radius. Needlessly complicated formulæ, involving higher or lower powers of the diameter, are thus replaced by the simple straight line. In the two bottom lines L and F, pertaining to the screwed portions of the fitting, which are a function of the pitch, the irregularities of slope are due to the fact already commented upon, that the pitch of the screw-thread for different diameters of tube proceeds by a medley of arbitrary jumps, instead of by a steady progression concordant with that of the tube diameter.

Similar scales, but much more elaborate, furnish the dimensions for the valves and other details in the construction of steam-heating apparatus. The leading example already given will suffice however as an illustration of the method.

It might seem that too much prominence has here been given to minor details, in dwelling at such length upon the screw threads and the exact proportions of tubes and fittings; but it can be unhesitatingly asserted that facility of construction, economy of material, and careful elaboration of detail, have been largely concerned in the establishment in America of an industry almost unknown in Great Britain, and of which the history has hitherto remained almost wholly unrecorded. Indeed the only book in the English language on heating by steam, until the publication in America of an elementary work a few months ago, was that of Robertson Buchanan, 1811–1814. Beyond the catalogues and circulars of manufacturers and dealers, little or no practical information has appeared in print; while a whole class of skilled workmen, and nearly forty years of experience have developed the practicability of the

system, and extensive usage has accustomed the entire American community to its attendant discomforts and annoyances.

Boilers.—Most frequently, and in particular for large apparatus, the steam for warming in America is supplied by one or more horizontal tubular or Seguin boilers set in brickwork. For a small extent of warming apparatus the vertical tubular boiler with internal fire box is possibly the one most usual. For medium and large apparatus the original Seguin boiler is adopted, which consists of a horizontal cylindrical shell, containing tubes in the lower half, with a steam dome on the top; the tubes are arranged in vertical and horizontal rows, not alternating or zigzag; and the best practice is to place a manhole in the front end over the firedoor, and beneath the tubes. The boiler is fired underneath, and the products of combustion return through the tubes from the back to the front, and then pass back again over the top of the boiler, which is covered in with brickwork: the temperature of this top flue rarely exceeds 400° Fahrenheit. The fuel used is anthracite coal, yielding 8 to 12 per cent. of ash; and when well supplied with air it evaporates from 8½ to 9 lbs. of water per lb. of coal. The boiler fittings, gauges, valves, pipes, &c., are such as are usual for boilers in England; only, as they are rarely made by engine-builders, but by the "steam-fitting" maker, it can be claimed for them that they are generally more sightly, as well as better and less expensive, than in England. Dampers and damper-regulators are in general use, the latter being constructed with elastic diaphragms arranged to be loaded for any desired temperature, and often with some automatic arrangement for changing the load to meet changes of temperature outside or inside the building warmed. Where the whole of the circulation through the warming apparatus is entirely closed from communication with the atmosphere, the boiler pressure is sometimes allowed to fall below atmospheric pressure to as low as 140° temperature of the steam.

One other boiler at least has successfully stood the severe test of competition with the Seguin boiler, namely that known as the Babcock and Wilcox water-tube circulating boiler, having a horizontal steam-drum or separator overhead. This boiler was tried by Mr. Nason in nearly its present form, but with smaller tubes, the joints of which failed as made by him.

Either of these boilers is practically safe from disastrous explosion. Out of the many thousands of Seguin boilers which have now been in use from one year to forty years, only two or three well authenticated instances have occurred of violent explosion, and none, so far as the author is aware, have involved loss of life. Of all the stationary boilers that are employed in the United States for any purpose whatsoever, perhaps one half are of the Seguin type. The deterioration of all heating-apparatus boilers is more serious from rusting in summer when idle than from wear in use. In fair service the duration of the shell of the horizontal tubular boiler is from thirty to forty years, at least where water is employed that does not produce scale or deposit. The wrought-iron flue-tubes cannot be considered to last more than seven or eight years, even with water of good quality.

The most serious defect in the Seguin boiler is the liability of the bottom plate just over the fire-bridge, to "come down" or bulge outwards, or to become blistered when strong firing is carried to the extent of burning 10 lbs. of coal or upwards per square foot of grate per hour. This accident is more likely to happen when the tubes are arranged zigzag in alternate rows, or are placed close together the circulation of the water between them being impeded in either case; and the provision of a large water-space in the bottom of the boiler, such as results from the practice already mentioned of placing a manhole in the front end beneath the tubes, tends to obviate the risk. Recently a hollow framing or slab, made of boiler-plate, filled with water and connected with the boiler by circulating pipes, has been substituted for the front plate of the furnace containing the fire-door, and thus forms a permanent wall, serving also as a saddle to support the front end of the boiler. The details of brickwork setting, flues, and chimney, present no unusual features as carried out in America.

There are, of course, innumerable varieties of boilers and settings. The

possible contortions of boiler-plates tubes, and castings, are far from being exhausted; and the economy claimed for each fresh device is generally in proportion to its complication. Just now there is a crop of devices in America for the more complete combustion of gases, after the plans generally of the late Mr. Charles Wye Williams, Assoc. Inst. C. E.; some of these, and others for burning such fuel as saw-dust, coal-dust, spent tan, &c., are as successful as they were thirty years ago.

For the fire-grates also of the boilers there is a revival of sundry shaking appliances; but for the American anthracite these schemes will speedily die out, since it resents the least disturbance after having been once ignited, and admits only the gentlest turning at bottom for allowing the ash to run out like sand from an hour-glass. An improved grate, however, made with wrought-iron fire-bars of square or triangular section, with their ends resting in bearings so that each bar separately can be rotated, seems really to have won favor for heavy firing, and must be mentioned with approval.

For the smaller sizes of warming apparatus there are several devices of magazines for holding a store of fuel, from which a continuous supply to the fire-grate is dragged down or allowed to run down upon the grate. Some of these magazines hold 4 to 8 tons of coal, which will last from one month to three months. It may be mentioned that American anthracite coal, having the strength of most English building stone, is got from the seam in massive blocks; these on reaching the pit bank are broken up into lumps, which for domestic use are screened to various sizes, ranging from as small as peas to as large as eggs, while the "furnace coal" for manufacturing purposes is sorted in still larger sizes.

THE ENGLISH MILE: ITS RELATION TO THE SIZE OF THE EARTH, AND TO ANCIENT METRICS.

By JACOB M. CLARK, C. E.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

THIS itinerary, on account of its lack of geographical correlation, and singular dimension, has evoked much interesting discussion, and been the means of bringing to the surface, under new aspects, a variety of important facts.

The reader is referred to a very instructive article in Vol. 25, p. 69, of this magazine, giving a full abstract of the views of M. Faye, as read before the Paris Academy, "On the Origin of the English Mile."

In that paper, the writer favors the view that the dimension is traceable to the survey of Eratosthenes, compared with that of Ptolemy; and, incidentally, that the surveys were conducted in terms of the Babylonian degree, and by implication, for the purpose of determining its length, or rather the subtense of one minute of arc; that the error in dimension arose partly from misapprehending the relative values of the stadia of different epochs, through disregarding the assumption that the computation of Eratosthenes was based on surveys made with the Egyptian foot* (0.27 m. = $10\frac{3}{5}$ inches), while the survey of Ptolemy was based on the Philetærian foot (nearly 0.36 m. = $14\frac{1}{6}$ inches).

Much additional light is thrown on the subject through a valuable contribution to these pages, from the pen of Prof. Mansfield Merriman ("The Shape and Size of the Earth," VAN NOSTRAND'S MAGAZINE, Vol. 22, pp. 53–62, 115–128, and 233–241). Reference is more particularly made to the different versions of the earth's circumference by ancient mathematicians on page 58. In the absence of direct evidence to the contrary, the results, the definition of amplitude observed by Eratosthenes, and the chronol-

* The Egyptian foot is uncertain. The dimension above given agrees very fairly with the ady of Malabar, the palmo of Malta, Messina, Naples, Sardinia, and Nice, the pied of Rouen, the stonecutter's schuh of Zug, and the miner's spanne of Prussia; dimensions varying from 10.265 to 10.570 inches.

The Philetæric foot is quoted by Alexander as equivalent to 13.893 inches.

ogy,* as given by Prof. Merriman, must be taken as clear and conclusive.

Hitherto, the moderns have regarded these statements as the results of successive experiments by the ancient geometers, to ascertain what has been supposed to be unknown to them—the length of the terrestrial degree, or, at any rate, the true circumference of the earth; and, on the face of it, the discrepancy is certainly glaring enough to justify the impression, and, at the same time, to suggest the theory both of serious errors in their work, and confusion among them as to the dimensions of the stadia used in the different surveys.

The true character and object of these operations can only be understood by reading them in the light of contemporaneous history and in view of the spirit of the times, aided by such knowledge of the actual dimensions made use of as can be obtained or fairly inferred.

It should be borne in mind that, as a rule, conquest has always involved more or less serious interference with the metries of the people. And, from the nature of the case, this was a prominent feature of state policy among the ancients. So long as a subjugated, but powerful and intelligent people retained the use of their traditional measures, cherished by the philosophers, and indissolubly connected with the mysteries and service of the temple, their complete subjection would be a matter of doubt. But the perils involved in the sudden and arbitrary overthrow of entire systems, were generally, in fact, sought to be avoided by modifications—compromises, under more or less specious and flattering pretexts.

Now as to the measures—the stadia probably used—and the mode of reckoning:

The Greek stadium was $\frac{1}{8}$ of their mile of 1,000 paces, or double parade-steps; value = $604\frac{8}{8}$ feet. The Romans had practically the same mile. Elsewhere, the stadium was 100 fathoms, and the fathom, generally, 3 cubits. But the Hebrew fathom was 4 cubits.

The Egyptian, Phenician, and Persian cubit was $\frac{1}{100}$ of the schœnus, which was equivalent to $145\frac{92}{100}$ English feet. The Hebrew cubit was $\frac{1}{80}$ of the schœnus.

The Babylonian cubit was apparently the subtense of 1° on a radius of 100 duodecimal feet, or 1,200 inches. Its value would depend on that of the inch. A form of it appears in Egypt, with some uncertainty as to the date of its introduction, under the name of the royal cubit. The Turin and Nilometer cubits, so called, are versions of it. The dimension was not far from 1.75 feet. A slight modification, to be understood further on, would place it at 1.75104 English feet.

There would result:

The Egyptian stadium = $437\frac{76}{100}$ English feet.
The Hebrew stadium = $729\frac{6}{10}$ English feet.
The (supposed) stadium on fathom of 3 Hebrew cubits = $547\frac{8}{10}$ English feet.
The (supposed) stadium on fathom of 3 royal cubits = $525\frac{312}{1000}$ English feet.

Among the peoples concerned, the reckoning, for all general purposes, was purely decimal, except that the Babylonians had the duodecimal mixed up in their system, with alternations of 6 and 10.

To facilitate the view, the different results are arranged in the table in chronological relation with prominent epochs, and in connection with the above lengths of stadia:

Epoch.	Event, etc.	Earth's circumference in stadia.	Length of stad. English feet.
B. C. 538	Babylon taken by Cyrus.		
B. C. 525	Independence of Egypt destroyed by Cambyses.		
B. C. 340	ARISTOTLE.....	300,000	437.76
B. C. 332	Macedonian Conquest; end of the Pharaohs.		
B. C. 250	ARCHIMEDES.....	300,000	437.76
B. C. 230	ERATOSTHENES....	250,000	525.312
B. C. 146	Greece made a Roman province.		
B. C. 90	POSIDONIAS.....	240,000	547.20
B. C. 30	Cleopatra's death; end of the Ptolemies; Egypt becomes a Roman province.		
A. D. 170	PTOLEMY (in the reign of Marcus Aurelius).....	180,000	729.60

With these values of the stadium, the circumference in each case is 131,328,000 English feet.

* In a recent discussion (Trans. Am. Soc. C. E., vol. XI., p. 415), the writer, adopting this chronology, inadvertently placed Aristotle 200, instead of 100, years earlier than Archimedes.

By Clarke's elements of 1878, as quoted by Prof. Merriman, the mean circumference is 131,331,455 English feet.

The skill and accuracy of ancient astronomers is strikingly illustrated by the survey of Almamoun, in Mesopotamia, in the 9th century, referred to for illustration by both Prof. Merriman and M. Faye. Taking the Arabian mile (palpably a version of Ezekiel's 500 reeds) at Haswell's quotation, 2,146 yards, with the Professor's statement of the result, $56\frac{2}{3}$ miles to the degree, the circumference is $2{,}146 \times 3 \times 56\frac{2}{3} \times 360 = 131{,}335{,}200$ feet, a trifle above the ancient and modern, in a total disagreement as to the whole circumference of less than a mile and a half.

Both Egypt and Mesopotamia are fairly situated for apprehending the mean circumference by meridian observations.

Leaving aside, for the moment, the above suggested adjustment of the royal cubit, the question arises pretty distinctly whether the most promising theory to square with all the facts may not, after all, be something like this:

(1.) At the very earliest assignable epoch, the mean circumference of the earth, and consequently its radius, were known with astonishing precision. Under a very perfect system of geometry, the metrics of the ancient leading nations were founded on this knowledge. The opinions of Aristotle and Archimedes were derived from this source, through Egypt.

(2.) After the Macedonian conquest, it became apparent that, by the breaking up and commingling of nationalities, the multiplicity of units was inconvenient and perplexing. The Mosaic and Babylonian cubits were in collision. The "cubit and a hand-breadth" of Ezekiel, by this time widely diffused and popularized, differed from the 2-feet rule by about an inch. The Egyptian and Persian cubit was becoming confounded with the Indian* cubit of 18 inches. And the Greek measures were a new element of discord.

Eratosthenes was charged by Ptolemy Philadelphus with the work of reform. To satisfy the prevailing preferences for the decimal method, and at the same time strike a reducible mean among the cubits, an itinerary was invented which should be an even decimal of 4 terrestrial great circles. It is more than probable that the survey of Eratosthenes was simply to test the correctness of the ancient standards, and fix the adjustment of the royal cubit. The circumference was already known, according to the Egyptians, and if their account proved correct, the *relation* was apparent beforehand. The royal cubit would have to be $\frac{6}{7}$ of the Egyptian $= \frac{24}{25}$ of the Mosaic $=$ 1.75104 English feet.

(3.) This unwieldy division of the circle, unfit for geography or astronomy, along with the strong preferences of the Egyptians, Persians, Hebrews and kindred races for the ancient measures, and their wide-spread traditional sympathy as against Babylonian methods, finally broke this system down. Accordingly, after the Roman supremacy was established, and in the reign of one of the later Ptolemies, Posidonias restored the Mosaic cubit, but in a 3-cubit fathom, so that his itinerary was decimally related to the hour angle. And so far from his survey being the worst measurement of a degree ever made, it serves to verify in a very lucid way the work of his predecessors. The aptness of his system, as an itinerary, is attested by the survival of the Turkish mile, through all vicissitudes, and of its correlative fathom, by dozens of analogues, in the islands and along the Mediterranean and in Central Europe. And it may seem significant to many that in the Apocalypse, written 186 years later, the division of the circle by 24 is paramount.

(4.) Finally, in the reign of Marcus Aurelius, a further attempt at unification seems to have been made, by re-instituting the Mosaic itinerary—the leuga of the ancient Gauls and the mile of Sardinia. This was the survey of Ptolemy. Possibly it involved some concession to the Egyptians, in that the schœnus became simply 20 fathoms, instead of $33\frac{1}{3}$ as by their ancient method or $26\frac{2}{3}$ as by that of Posidonias. It lacked, however, the key-note of Ezekiel's reform—radius to be the base of direct

* It is a peculiarity of the purely duodecimal method that, reckoning from the inch, it has no longer dimension than the 12-feet pole or "joktan." And, wherever it has taken root, this dimension, as well as its derivatives by bisection, the vulgar fathom, the yard, and the Indian cubit, as also the foot, when used as units, are, as a rule, but with occasional intermediaries of 3 and 6, reckoned upwards decimally. Its singular distribution—about the Mediterranean, among the islands, as Great Britain and Japan, and upon the salients of Africa and Asia—are strikingly suggestive of the maritime enterprise of busy Tyre.

and square measure, itinerary to be ruled by the division of the circle.

There seems to be little, either in the accounts we have, the necessities of the case, or the character of the rulers under whom these operations were conducted, to indicate that they were instituted for any other purpose than that above suggested—verification of the ancient work, and adjustment of the standard for the particular purpose in view at the time. Neither is it apparent that in any of them, the Babylonian degree was either used or its dimension sought (except possibly as to Ptolemy), or that the Egyptian foot, whatever it may have been, or the Philetærian foot or the Greek stadium were used or referred to at all, or that the geodetic work had any other fundamental base than the ancient public surveys of Egypt, familiar to the learned throughout the whole period of operations, and unquestionably made with the schœnus. And this view is confirmed by Eratosthenes' definition of amplitude, as quoted by Prof. Merriman—$\frac{1}{50}$ of 4 right angles — a decidedly decimal expression as well as by the general ancient preference for the decimal method.

The English mile, by its dimension, suggests with strong probability that it was at first either equal to 1751 yards, representing the survey of Eratosthenes, or else 1824, like the Turkish mile, the itinerary of Posidonias; and that it took its present form at the time the English forced the 36-inch yard into their land measure, by means of the invention of Gunter's chain. The former is the more probable of the two.

ELECTRIC POTENTIAL, ENERGY AND WORK.

By Lieut. BRADLEY A. FISKE, U. S. N.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

IN no study more than in that of electrical science is a clear comprehension needed of the terms used in the nomenclature, and of the laws under which the forces act. The purely experimental stage of its development has passed, and we have now reached a point where theory and practice must work hand in hand. A careful effort to apply its principles to its practical applications will discover the methods of securing effects by the most rapid and inexpensive means, and it is a well-known fact that the surprising results obtained in practice by many inventors are not due to happy inspiration or to experiment alone, but also to profound and laborious calculation.

To many, the terms heading this paper convey a very indefinite meaning, and their relation to the question of electric supply, lighting, etc., seem very remote. It can be shown, however, that they possess a most real and absolute signification, and that they bear directly upon nearly all the problems of electrical engineering.

The books tell us that electric potential means power to do electrical work, and that the difference of potential between two points is the number of units of work necessary to be done on a plus unit of electricity in order to move it from one to the other.

Work, as is well known, is the product of force by distance. In order to ascertain the relations between electrical and mechanical work, it has been necessary to devise units in terms of which both can be expressed.

In the system invented, known as the c. g. s. system, the unit of force is called the *dyne*, and means that force which, acting for one second on a mass of one gramme gives it a velocity of one centimeter per second. The unit of length is the centimeter; and the unit quantity of electricity assumed is that which, placed at a distance of one centimeter from a similar and equal quantity, repels it with a force of one dyne.

The dyne being the unit of force, and the centimeter the unit of length, the unit of work must be the dyne-centimeter. To this unit is given the name *erg*.

From the fact that charges of electricity of similar name repel one another, it is evident that to move a plus unit of electricity towards a plus charge necessitates the expenditure of work. Suppos-

ing the plus unit to be at an infinite distance from the plus charge, and therefore free from its influence, and then to be moved towards it, the number of ergs expended in getting it up to a certain point expresses the potential of that point. To move the unit to a point nearer the charge would clearly require a greater expenditure of work; in other words, the potential of the second point would be higher.

Now, it is obvious that to move any plus unit from the first to the second point would require an amount of work equal to the difference of the two amounts; so that, if this difference were one erg, the difference of potential between the two points would be one.

If, on the other hand, instead of raising a quantity of electricity from a lower to a higher potential, we allow it to fall from a higher to a lower, it will, in accordance with the law of conservation of energy, perform exactly the same work as was expended upon it in raising it, through the same difference of potential.

We see at once that this is the case with which we have to deal in computing the electrical work done in any circuit, for there we find units of electricity falling through units of potential. The units (called electro-magnetic units) are different from those thus far considered (called electro-static), but the same laws evidently apply.

An electro-magnetic unit of quantity is that carried in one second by a current of such a strength that, if a length of it equal to one centimeter be bent into an arc having a radius of one centimeter, it will act with a force of one dyne on a unit magnet pole placed at its center. Now, this unit bears to the electro-static unit the ratio

$$LT^{-1};$$

but as the electro-magnetic difference of potential bears to the electro-static the ratio

$$\frac{1}{LT^{-1}}$$

one electro-magnetic unit performs just one erg of work in falling through one electro-magnetic unit of potential.

Therefore, if we know the strength of current passing through any circuit or part of a circuit, and the potential through which it falls in so passing, the work performed in one second is clearly

$$CE,$$

or, as sometimes written,

$$C^2 R \text{ or } \frac{E^2}{R}.$$

But, as these units are manifestly too small, larger and more practical ones are used. Of these, the volt equals 10^8 electro-magnetic units of potential, and the ohm equals 10^9 electro-magnetic units of resistance. Necessarily, therefore, the practical unit of quantity is 10^{-1} times the electro-magnetic unit.

Therefore, if in measuring with our practical instruments, we find a strength of current C′ and a fall of potential E′, it is obvious that the work done in one second is in ergs,

$$C'E' \times 10^7.$$

Knowing, as we do, that one horse-power means an energy of 746×10^7 ergs in one second, the horse-power performed is, of course,

$$\frac{C'E' \times 10^7}{746 \times 10^7} = \frac{C'E'}{746}$$

or, as often written,

$$\frac{E' \times 44.24}{R \times 33.000}$$

As it is potential which does the work of driving the current through the resistances of the circuit, it follows that less potential is expended in any circuit in overcoming the small resistances than the great ones. From this it follows again that, in any simple circuit, the fall of potential in different parts is proportional to the resistances of those parts; and also that, as no generator, whether battery, accumulator, or machine is a perfect conductor of electricity, we must not neglect to take its resistance into consideration. This is frequently neglected, and great mistakes sometimes ensue in consequence. The Electrical Power Storage Co., in England, gave out recently that the number of their accumulators necessary to feed a given number of incandescence lamps could be calculated by simply dividing the potential needed for one lamp by the electro-motive force of one cell. Though plausible upon its face, this rule is clearly incorrect, for the rea-

son that all of the electro-motive force generated is not furnished to the external circuit, but only the fraction thereof, represented by the quotient of the external resistance divided by the generator's resistance plus the external resistance. Evidently, when a great many lamps in multiple arc are fed by a number of accumulators in series, the distribution of potential is far from being the same that it is when only a few are supplied. Suppose we wish to know how many accumulators, having each an electro-motive force e, are necessary to supply n incandescence lamps, having each a resistance r' when hot; the potential necessary for each lamp being e'.

The whole current necessary, then, is

$$C = n\frac{e'}{r'}.$$

Suppose the current which each cell is capable of furnishing most economically to be c. Then we must have such a number of series of cells that the whole current shall be c. Letting $y =$ this number—

$$y = \frac{C}{c}.$$

Let x = No. cells in each series,
" r = resistance of each cell,
" R = " " conductors.

Then

$$C = \frac{xe}{\frac{xr}{y} + \frac{r'}{n} + R} = \frac{xeny}{nxr + r'y + Rny}.$$

$$Cnxr + Cr'y + CRny = xeny$$

$$x(eny - Cnr) = Cr'y + CRny.$$

$$\text{But } C = \frac{ne'}{r'} \therefore n = \frac{r'C}{e'}$$

$$\therefore x(ey - Cr) = e'y + \frac{R}{r'}ne'y.$$

$$x = \frac{e'y + \frac{R}{r'}ne'y}{ey - Cr.}$$

In case R is very small as compared with r', this formula may be written—

$$x = \frac{e'y}{ey - Cr.}$$

It will be noticed that we have considered this as a simple circuit by taking the joint resistance of the lamps and conductors as a single external resistance.

In calculating the fall of potential in any branch of a divided circuit, we have to deal with somewhat different conditions. When two or more branches divide and reunite at common points, the difference of potential at the ends of each branch, instead of being that due to the resistance of that branch, is only that due to the joint resistance of all the branches. When lamps or motors are supplied in multiple arc, however, we cannot say that the difference of potential supplied to each lamp is that due to the joint resistance of all the lamps, even if all are burning, for the obvious reason that each lamp does not get its wires direct from the generator, and that all are not at the same distance therefrom. For this reason, the potentials furnished all the lamps are not equal, but vary with their distances from the generator, or rather with the resistances of the mains between the generator and the points from which the lamps draw their supply. Clearly, a lamp near the generator will get the benefit of nearly the whole potential furnished to the circuit, for the reason that it takes one wire from very near the positive pole of the generator, and the other from very near its negative pole; while the case of the lamp farthest away is manifestly different. This lamp takes one wire from a point where the potential is less than at the positive pole, and takes the other wire from a point where it is greater than at the negative pole. In order that the difference of potential furnished the lamps may vary as little as possible, both from this cause and by reason of adding lamps to or subtracting them from the circuit, it is clear that the resistances of the mains and the machines should be as small as possible.

Remembering the formula

$$CE,$$

and remembering, too, that in any simple circuit, the fall of potential is proportioned among the resistances, the formulas follow for the *electrical* efficiency of machines—*i. e.*, the ratio of the energy in the external circuit to that generated.

The most simple case is that of the series dynamo. As the current, C, is the same in the external circuit, the

armature and the magnet coils, the efficiency F is clearly

$$\frac{r}{a+m+r}$$

where r is the resistance of the external circuit, a that of the armature, and m that of the magnet coils.

In the case of the shunt dynamo, the current in the armature is equal to the sum of the currents in the external circuit and the magnet coils, supposing shunt taken direct from brushes.

Let A=current in armature, then current in main circuit=

$$A\frac{m}{m+r}$$

and the whole resistance of the circuit

$$R=\frac{mr}{m+r}+a$$

$$\therefore F=\frac{\left(A\frac{m}{m+r}\right)^2 r}{A^2\left(\frac{mr}{m+r}+a\right)}=\frac{m^2 r}{a(m+r)^2+m^2 r+mr^2}$$

The case of a dynamo whose field magnets are wound partly in shunt and partly in series is very similar.

It is obvious that the formala for the proportion of energy consumed in the external circuit plus that consumed in the series coils (supposing the shunt to be taken direct from the brushes) may be gotten from the above formula by simply substituting r' for r, in which case r' equals the added resistances of external circuit and series coils.

$$\therefore F'\frac{m^2 r'}{a(m+r')^2+m^2 r'+mr'^2}$$

Letting r, as before, equal the resistance of the external circuit alone, we have

$$F : F'=r : r'$$

$$F=\frac{Fr}{r'}$$

$$\therefore F=\frac{m^2 r}{a(m+r')^2+m^2 r'+mr'^2}$$

In the case of transmitting power from one point to another, the amount of energy consumed in the wires becomes a most important consideration. This energy appears in the form of heat, which is not only lost for all practical purposes, but which has the further effect of raising the resistance of the wires, which, in turn, causes the absorption of more heat.

Evidently, as we are to give power to the wire, we wish to know in what form it is most economical to give it, in order to reduce this loss in transitu to a minimum. Power being composed of the two factors C and E, is it better to make C and E large? A casual inspection of the formula $\frac{E^2}{R}$ might lead to the hasty conclusion that it were better to generate as low an electro-motive force as possible. But this conclusion would manifestly be incorrect, for the reason that this would necessitate a large C in order to produce to requisite power; and we all know that heat in any circuit varies as C^2. The formula, it need not be said, however, is correct, for here E means not the whole electro-motive force, but only that fraction thereof represented by the ratio of the external to the whole resistance. Thus we see that we must get R as small as possible, because, although E diminishes in the same ratio, yet E enters by its square. We also see that the whole resistance as compared with R must be great. In other words, we must use a machine with considerable resistance generating a high electro-motive force, and conductors of small resistance. Thus we clearly get a small C.

It is to be observed, however, that, in cases of power distribution, other considerations may have weight.

It may be desired, for instance, to transmit power during the day by the same company which supplies incandescence lamps by night. In this case it would probably be found more economical to use to same wires and to supply motors in multiple arc, than to have a separate system supplying them in series. The greater safety of a low tension current is, moreover, a most powerful consideration.

In applying the formulas

$$C\,E\frac{E^2}{R}\text{ and }C^2R,$$

care must be taken to remember that as C indicates the quantity of electricity passing in one second, these expressions indicate not work, but the amount of

work performed in one second. In order to get the work, we must either multiply the expression representing the energy by the time during which that energy was furnished, or else we must find the number of coulombs passing in the time. Thus we see the rationale of the Edison meter. The indicator shows the number of coulombs which have passed; and as the potential is kept nearly constant, this indication furnishes a measure of the work which has passed through the subscriber's wires. In other words, the subscriber does not pay for the amount of electricity furnished, but for the work performed.

In conclusion, it would seem pertinent to suggest that it is not well to have the same unit, the erg, for both energy and power. Almost every issue of the English "Electrical Review" contains communications from persons, who, from that cause alone, seem to hopelessly confound electrical energy and power.

FAST TRAINS IN EUROPE AND AMERICA.

By Mr. A. L. ROTCH.

Read before the E. M. E. Society of the Massachusetts Institute of Technology.

I SHALL first consider American trains, taking into account the acceleration which has occurred within the last few years. For England, my data are brought down to 1880, and on the Continent it is as old as 1878; but, generally speaking, the figures are sufficiently accurate to-day to admit of comparisons.

AMERICA.—The fastest trains in the United States are in the East, and are those between Jersey City and Philadelphia, Boston and New York, and one from New York to Albany. In selecting representative trains I have given preference to those which maintain the highest speed for the longest time. Probably the fastest train in this country, and one which is little below the speed of the fastest English trains for the same distance, is the Philadelphia express, on the Pennsylvania Railroad, which leaves Jersey City at 4.08 p.m., and makes the run of 88.4 miles in 1 hour 52 minutes, including three stops, or at the rate of 47.8 miles per hour. Jersey City to Germantown Junction, 84.2 miles, is run in 1 hour 41 minutes, including one stop, or 50.5 miles per hour. I timed last winter the fastest train east, whose schedule time is 1 hour 59 minutes. It was then, however, 6 minutes late, and as it was bulletined "on time," perhaps it seldom does better. The train consisted of six cars, one of them a Pullman. A stop was made at Germantown Junction, and it was some time before a high speed was attained. The fastest miles were run in 63 seconds each, or 57 miles per hour; 21 miles were made in $25\frac{3}{4}$ minutes, or 49 miles per hour. The 84.2 miles from Germantown Junction to Jersey City were run in 1 hour $52\frac{1}{4}$ minutes=45 miles per hour, including slackening through several towns. It may here be said that the Pennsylvania road-bed equals that of any foreign road I have seen, while the freedom of the train from oscillation and vibration was remarkable. This train used to run in five minutes less time, and the fast train west was two minutes quicker when started a few years ago, so perhaps it was found impossible to maintain these higher speeds. The Bound Brook route of the Philadelphia and Reading Railroad has a train from Philadelphia to Jersey City, 89.4 miles, in 1 hour 57 minutes, including five stops, or 45.8 miles per hour. Wayne Junction to Jersey City, 85.1 miles, is run in 1 hour 47 minutes, including three stops, which is 47.7 miles per hour. The 4.30 p.m. train from New York to Boston *via* Springfield, is the fastest between the two cities, taking 6 hours 3 minutes for the 234 miles. I timed the train last September when it was on time. The engines were of the ordinary type, with about 5 feet 6 inch drivers, and the train consisted of six cars, two of them being parlor cars. On the New Haven Division, the fastest runs were 1.90 miles in two minutes=57 miles per hour; 1.73 miles in one minute 55 seconds=54.5, and three miles in 3 minutes 15 seconds=53 miles per hour. Owing to the Connecticut law requiring a stop at every drawbridge, the time for

the 73¼ miles between New York and New Haven, including six stops, was 1 hour 54¼ minutes, or only 38.5 miles per hour. The 62 miles between New Haven and Hartford were made in 1 hour 30¼ minutes, including two stops, or 41.5 miles per hour; average speed while running, 43.8 miles per hour. On the Boston and Albany, 96 miles (Springfield to junction of Brookline Branch, Boston) were run in 2 hours 20½ minutes, including three stops, or 41 miles per hour; running speed 43.2 miles per hour, including stops. The time for the trip of 234 miles was 5 hours 30 minutes, and the speed 42.5 miles per hour. The Shore Line express makes its run to Stonington, 93 miles, in 2 hours 1 minute including a stop at Providence, or over 46 miles per hour. As long ago as 1878 the *Railroad Gazette* said: "For over two years a daily train has been run over the 43½ miles between Boston and Providence in one hour, including a dead stop at the crossing, a 6-mile grade over Sharon hill of 35 feet to the mile, a slowing up at Mansfield and through Pawtucket, over a mile, and a slacking over three bridges. The engines have 5½-foot drivers and 17-inch cylinders. The train is a baggage and smoker, and there are sometimes four 24-ton passenger and a 32-ton Pullman." In February, 1881, I timed this train as follows: Left crossing 1.03 p.m. Beyond Foxboro' the speed was a mile in 63 seconds, or 57 miles per hour; 2.48 miles were run in 2 minutes 40 seconds=55.5 per hour. Before reaching Pawtucket, 14.9 miles had been run in 17½ minutes=54.5 miles per hour. Providence was reached at 1.59 p.m.=45 miles per hour. From the crossing the speed was 47 miles per hour. Last June this train was quickened beyond Providence as above. The fastest train on the New York Central Railroad is the Chicago and Lake Shore special, which runs to Albany, 140 miles, without a stop, at the rate of 49.5 miles per hour. The first 58¾ miles are made in 1 hour 28 minutes. These figures should, I think, be sufficient to show that regular trains in this country seldom run a mile in 60 seconds. The fastest run which I have made here was on the New York and New England Railroad the other day, when, on a down grade beyond Bolton Notch, 4 miles were covered in 4 minutes 10 seconds, or 58 miles per hour. In England I have been faster, and it is said, on authority, that it is not uncommon for English express trains to make 60 or 70 miles per hour when running on their schedule time. It has been said that it is impossible for an engine with a 5½-foot driver to make a mile in a minute. Mr. Le Van gives the greatest piston speed as about 1,200 feet per minute, which would give about 59 miles per hour with such a wheel. On the Pennsylvania the fast train engines had two pair of drivers 68 inches in diameter and had to make 300 revolutions to make a mile a minute. Coupled drivers are said to be dangerous at high speeds from the liability of the parallel-rods to break, and such accidents have recently happened. Still, in late years, large coupled wheels have been much used on English and continental express engines. When in Europe during 1877–78, I looked up the fastest trains in the several countries, and as I traveled by some of them I further noted their maximum speeds. A few general remarks about European railways may be interesting. The continental railways are partly controlled by the governments. They are well built, with curves of large radius. Instead of cars there are carriages, divided into compartments holding six or eight persons. They do not, as a rule, have bogie trucks, but their small wheel-base enables them to pass round the curves. There are three or four classes of passengers. Generally, first and second class are carried by the express trains, but in France only first-class are taken, and a higher class are charged. The locomotives often have only a single pair of drivers, 7 feet or so in diameter. In England 6½-feet driving-wheels are quite as common as 5½ feet with us, and some engines on the Great Western and Great Northern railways have wheels 8 or 9 feet in diameter. There is much dispute as to the merits of single *versus* coupled engines, but I noticed at the Paris Exposition that all the French express engines shown had their driving-wheels coupled. Many of the modern engines have cabs and bogie trucks in front. Distances are given on the Continent in kilometers (0.62 mile each), which have been converted into miles for the sake of comparision here.

SWITZERLAND AND RUSSIA.—There are no trains exceeding 27 miles per hour in those countries, which are, therefore, not considered. In Belgium, trains travel as fast as 42 miles per hour, but these are generally through trains between France and Germany, and may, therefore, be classed with the trains of those countries.

ITALY.—The only fast train is the mail which makes the long run from Bologna to Brindisi, 472 miles, in 14 hours 55 minutes, which, including three stops, is 31.5 miles per hour. The train is largely due to English enterprise. It carries the English mails, and takes only through passengers.

FRANCE.—Many of the French expresses are fast. I believe the fastest long distance run is on the Orleans line, from Paris to Bordeaux, where 359 miles are run in 9 hours 10 minutes, including 17 stops, an average of over 39 miles per hour. Allowance being made for these, the average running speed is 42.5 miles per hour. I do not think this performance is surpassed in America. From Paris to Marseilles is 536 miles, which used to be covered in 15 hours 25 minutes, at the rate of 34.8 miles per hour, including 13 stops made. The time has since been reduced to 15 hours only. The expresses on the Northern Railway run from Calais to Paris, 184½ miles, at 36.7 miles per hour, including seven stops, or at over 39 miles per hour, while running. A run of 27 miles is made at the rate of 45.5 miles per hour. I traveled by one of these trains and found the oscillation of the short carriages tremendous. On the Eastern Railway, the schedule time for a run of 47¼ miles is 1 hour 5 minutes, and on the Western Railway, I timed the express from Paris to Dieppe, running at its usual rate of speed, 16¼ miles in 20 minutes 20 seconds, or 47.7 miles per hour.

GERMANY.—Taking Germany and Austria together, the fastest trains are found in North Germany. Undoubtedly the fastest train is from Berlin to Hanover, on the Magdeburg-Halberstadt Railway 152½ miles in 3 hours 48 minutes, including three stops=51.7 miles per hour. Deducting these, the speed is 44 2 miles per hour. This beats the fast train from New York to Albany, which is the fastest train in America for that distance. A run of 93¼ miles is made by the German train in 2 hours 8 minutes=44 miles per hour. I timed the part from Berlin to Stendal. Some miles were run at 52 miles per hour; 62¼ miles were done in 1 hour 30 minutes, including a stop. Berlin to Cologne by this route is 363 miles, accomplished in 9 hours 41 minutes=37.5 miles per hour. On the Berlin-Potsdam-Magdeburg Railway the fast express I have timed 24¾ miles in 30 minutes=48.7 miles per hour and 50 miles in 1 hour 9 minutes=43.5 miles per hour. This forms part of the through line between Berlin and Paris, and is, perhaps, the best example of a very long run abroad. The distance is 668 miles, done in about 22¼ hours, or at the rate of about 30 miles per hour, notwithstanding the fact that three countries are traversed with attendant custom-house formalties at the frontiers. Thus it will be seen that the speeds of the fastest continental trains about equal our own, except in the case noted. For very long runs nothing there equals the speed of the New York-Chicago trains, which average some 35 miles per hour for over 900 miles.

ENGLAND.—In England, however, we are beaten both in number and speed of fast trains. Mr. Le Van says the average speeds are 20 per cent. greater and the loads 25 per cent. less. Where there are so many rival trains running at nearly the same speeds, it is difficult to name the fastest. The "Flying Scotchman" (*via* the Great Northern) runs to Edinburgh, 397 miles, in 9½ hours, including stoppages, at nearly 42 miles per hour. The Great Western has a 7-feet gauge, and formerly its express went from London to Bristol, 118¼ miles, in two hours =59 miles per hour. This train, known as the "Flying Dutchman," is still, perhaps, the fastest train in the world, though its speed has been much reduced. The 118¼ miles are now made in 2 hours 36 minutes=45.3 miles per hour, including two stops amounting to 11 minutes. Excluding these, the speed is 49 miles per hour. London to Exeter, 194 miles in 4¼ hours, exclusive of 20 minutes' halt on the way, is 49.5 miles per hour. London to Swindon, without stopping, 77¼ miles in 1 hour 27 minutes, is 53.3 miles per hour. The Leeds summer expresses on the Great Northern make the

186¼ miles in 3¾ hours=49.4 miles per hour, including two stops amounting to 8 minutes. London to Grantham, 105½ miles, in 2 hours 2 minutes=51.7, and Grantham to Wakefield, 70¼ miles, in 1 hour 17 minutes=54.7 miles per hour. This last is claimed to be the fastest run in the world. My timing of the Scotch express, the fastest train from London on the North-Western Railway, may be interesting. Before reaching Rugby 76½ miles had been traversed in 1 hour 35½ minutes=48 miles per hour. Crewe, 158 miles, was reached in 3 hours 39 minutes (beating the best Continental time for that distance), and Liverpool, 202 miles, in 5 hours, including six stops=45 miles per hour. A few miles were run in 61 seconds, which is at the rate of 59 miles per hour.

If the speed of American trains continues to improve during the next few years as much as it has recently, we may soon hope to outstrip England, where the maximum has been apparently reached. In fact Mr. Le Van has prophesied that within five years the distance from Philadelphia to Jersey City will be accomplished in one hour, but this seems rather too much to expect, especially on a road running through such a populous district.

HYDRAULIC TABLES BASED ON KUTTER'S FORMULA.

By P. J. FLYNN, C. E.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

SECOND PAPER.

EGG SHAPED SEWERS.—INTERNAL DIMENSIONS.

Depth of vertical diameter is 1.5 times the greatest transverse diameter; that is, the diameter of top of arch.

Let D=greatest transverse diameter, that is the diameter of top or arch=$\frac{2H}{3}$, then

H=depth of sewer or vertical diameter=1.5D.

B=radius of bottom or invert=$\frac{H}{6}$.

R=radius of sides=H.

By reference to column $c\sqrt{r}$ in Tables 3 and 4 it will be seen that the mean velocity of an egg-shaped sewer flowing two-thirds full is always *greater* than that of the mean velocity of same sewer flowing full. When the slopes are equal, columns $c\sqrt{r}$ and $ac\sqrt{r}$ give a ready means for comparing velocities and discharges.

APPLICATION AND USE OF THE TABLES.

To find the velocity and discharge in an egg-shaped sewer.

Example 10.—An egg-shaped sewer 7 feet by 10 feet 6 inches has a slope of 6 feet per mile. What is its velocity and discharge flowing full, flowing two-thirds full depth and one-third full depth?

A slope of 6 feet per mile is equal to 1 in 880, opposite to which in Table 2 the value of $\sqrt{s}$ is found to be=.03271.

In Tables 3, 4 and 5 opposite a transverse diameter of 7 feet find the values of $c\sqrt{r}$ and $ac\sqrt{r}$ and substitute them and also the value of $\sqrt{s}$ above found in formula (1) $v=c\sqrt{r}\times\sqrt{s}$.

" (5) $Q=ac\sqrt{r}\times\sqrt{s}$ and we get the following:

Full depth. $\begin{cases} v=160.2\times.03371=5.4 \text{ feet per second.} \\ Q=9015.7\times.03371=303.9 \text{ cubic feet per second.} \end{cases}$

Two-thirds depth. $\begin{cases} v=169.6\times.03371=5.72 \text{ feet per second.} \\ Q=6283.5\times.03371=211.8 \text{ cubic feet per second.} \end{cases}$

One-third depth. $\begin{cases} v=127.9\times.03371=4.31 \text{ feet per second.} \\ Q=1779.4\times.03371=59.98 \text{ cubic feet per second.} \end{cases}$

To find the dimensions of an egg-shaped sewer to replace a circular sewer.

Example 11.—A circular sewer 5 feet diameter and 4800 feet long has a fall of 16 feet. It is to be removed and replaced by an egg-shaped sewer with a fall of 8 feet whose discharge flowing full shall equal that of the circular sewer flowing full? Give dimensions of egg-shaped sewer.

A fall of 16 in 4800=1 in 300 and in Table 2 the $\sqrt{s}$ corresponding to this is .057735. In Table 1 opposite 5 feet diameter the value $ac\sqrt{r}$ is 2272.7, substitute this value and also the value of $\sqrt{s}$ in formula (5) $Q=ac\sqrt{r}\times\sqrt{s}$ and we have $Q=2272.7\times.057735=131.21$ cubic feet per second, the discharge of the circular sewer. The egg-shaped sewer is to have a fall of 8 in 4800=1 in 600, and in Table 2 the equivalent $\sqrt{s}$ is 040825, substitute this value and also the discharge found above in

formula (7) $ac\sqrt{r}=\frac{Q}{\sqrt{s}}=\frac{131.21}{.040825}=$ 3213.9.

In Table 3, the nearest value of $ac\sqrt{r}$ to this is 3353 opposite a transverse diameter 4 feet 10 inches, therefore the egg-shaped sewer is to be 4 feet 10 inches by 7 feet 3 inches.

To find the diameter of a circular sewer whose discharge flowing full shall equal that of an egg-shaped sewer flowing one-third full depth.

Example 12.—Find the diameter of a circular sewer whose discharge flowing full shall equal that of the egg-shaped sewer in last example flowing one-third full the slope being the same in each.

In Table 5 and opposite transverse diameter 4 feet 10 inches the value of $ac\sqrt{r}=657.53$.

In Table 1 the value of $ac\sqrt{r}$ nearest to this is found to be 661.77 opposite a diameter of 3 feet 2 inches, which is the diameter of the circular sewer required.

To find the diameter of a circular sewer whose velocity flowing full shall equal that of an egg-shaped sewer flowing one-third full depth.

Example 13.—What is the diameter of a circular sewer whose mean velocity flowing full shall equal that of an egg-shaped sewer 4 feet by 6 feet flowing one-third full, the grade in each being the the same?

In Table 5 and opposite the transverse diameter 4 feet the value of $c\sqrt{r}=86.61$.

In Table 1 the value of $c\sqrt{r}$ nearest to this is 87.15, opposite diameter 3 feet 4 inches, which is the diameter of the circular sewer required.

To find the dimensions and slope of an egg-shaped sewer flowing full, the mean velocity and discharge being given.

Example 14.—An egg-shaped sewer flowing full is to have a mean velocity not greater than 5 feet per second, and is to discharge 108 cubic feet per second. What is size and slope?

By formula (6) $a=\frac{Q}{v}$ substitute values of Q and v given and $a=\frac{108}{5}=21.6$ square feet.

In column 2 of Table 3 the nearest area to this is 21.556 opposite the transverse diameter 4 feet 4 inches, therefore the sewer required is 4 feet 4 inches by 6 feet 6 inches. At the same time the value of $ac\sqrt{r}$ opposite 4 feet 4 inches diameter is found equal to 2501.4, substitute this and also value of

formula (8) $\sqrt{s}=\frac{Q}{ac\sqrt{r}}=\frac{108}{2501.4}=$ .043176, and in Table 2 the nearest value of $\sqrt{s}$ to this is .043193 opposite the slope of 1 in 536, which is slope of sewer.

The diameter and slope of a circular sewer being given, to find dimensions and slope of an egg-shaped sewer whose discharge flowing two-thirds depth shall equal that of the circular sewer flowing full and whose velocity at same depth shall not exceed a certain rate.

Example 15.—A circular sewer 6 feet in diameter and with a slope of 1 in 600 is to be removed and to be replaced by an egg-shaped sewer whose discharge flowing at two-thirds of its full depth shall be equal to that of the circular sewer flowing full and whose mean velocity at the same two-thirds depth shall not exceed 5 feet per second? Give dimensions and slope of egg-shaped sewer.

In Table 1 and opposite 6 feet diameter the value of $ac\sqrt{r}$ is 3702.3, and in Table 2 opposite 1 in 600 the value of $\sqrt{s}$ is .040825, substitute these values in formula (5) $Q=ac\sqrt{r}\times\sqrt{s}$ and we get

$Q=3702.3\times.040825=151.15$ cubic feet per second, the discharge of the circular sewer. Now substitute this discharge and the velocity above given 5 feet per second in formula (6) $a=\frac{Q}{v}$ and

we get $a=\frac{151.15}{5}=30.23$ square feet, the area at two-thirds depth of the egg-shaped sewer. In column 2 of Table 4 the nearest area to this is 30.317 opposite a transverse diameter of 6 feet 4 inches, therefore the dimensions of egg-shaped sewer are 6 feet 4 inches by 9 feet 6 inches.

At the same time take out the value of $ac\sqrt{r}$ opposite 6 feet 4 inches, which is 4811.9. Substitute this and also the value of Q found in formula (8)

$$\sqrt{s}=\frac{Q}{ac\sqrt{r}}=.031412$$

and this not being found in Table 1, square each side and

$$s=.0009867,$$

and $\frac{1}{.0009867}=1013$ nearly, therefore the slope of egg-shaped sewer is 1 in 1013 and its size 6 feet 4 inches by 9 feet 6 inches.

To find the dimensions and grade of an egg-shaped sewer to have a certain discharge flowing full, and whose mean velocity shall not exceed a certain rate when flowing two-thirds full depth.

Example 16.—An egg-shaped sewer is to discharge 110 cubic feet per second flowing full and its mean velocity flowing two-thirds full depth is not to exceed 5 feet per second? Find its dimensions and slope.

As a first approximation assume the velocity flowing full at 5 feet per second, then $\frac{110}{5}=22$ square feet the area of egg-shaped sewer flowing full, and in Table 3 opposite this area the transverse diameter 4 feet 4 inches is found. Now with this diameter

the value of $c\sqrt{r}$ full depth $=116.0$

the value of $c\sqrt{r}$ two-thirds depth $=123.1$

therefore we may assume that the velocity of sewer flowing full is for 4 feet 4 inches, transverse diameter about 6 per cent. less than when flowing two-thirds full, that is, assuming the velocity at two-thirds depth 5 feet per second, the velocity at full depth will be about 4.7 feet per second. Substituting this velocity and also discharge in formula (6) $a=\frac{Q}{v}=\frac{110}{4.7}=23.4$ the area of egg-shaped sewer flowing full. In Table 3 the transverse diameter opposite this is 4 feet 6 inches, which is the diameter required of the egg-shaped sewer. At the same time that diameter is found look out the value of $ac\sqrt{r}$ which is 2770, substitute this in

formula (8) $\cdot\ \sqrt{s}=\frac{Q}{ac\sqrt{s}}$

$$=\frac{110}{2770}=.039711.$$

In Table 2 the $\sqrt{s}$ nearest to this is .039715 opposite a slope of 1 in 634, therefore the dimensions of egg-shaped sewer are 4 feet 6 inches by 6 feet 9 inches and its slope 1 in 634.

Now in Table 4 the value of $c\sqrt{r}$ opposite transverse diameter of 4 feet 6 inches is 126.3, substitute this and also value of $\sqrt{s}$ above found in

formula (1) $v=c\sqrt{r}\times\sqrt{s}$ and we have $v=126.3\times.039711=5$ feet per second, the mean velocity of sewer flowing two-thirds full.

TABLE 2—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=side of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
2640	.000378787	.019463	2280	.000438597	.020943
2600	.000384615	.019612	2240	.000446429	.021129
2560	.000390625	.019764	2200	.000454545	.021320
2520	.000396825	.019920	2160	.000462963	.021517
2480	.000403226	.020080	2120	.000471698	.021719
2440	.000409836	.020244	2080	.000480769	.021927
2400	.000416666	.020412	2040	.000490196	.022140
2360	.000423729	.020585	2000	.000500000	.022361
2320	.000431034	.020761	1980	.000505050	.022473

TABLE 2 *(Continued)*.—GIVING VALUES OF s AND $\sqrt{s}$.

h = sine of slope = fall of water surface (h) in any distance (l), divided by that distance = $\frac{h}{l}$.

Slope 1 in	s = sine of slope.	$\sqrt{s}$.	Slope 1 in	s = sine of slope.	$\sqrt{s}$.
1960	.000510204	.022588	1080	.000925926	.030429
1940	.000515464	.022704	1070	.000934579	.030571
1920	.000520833	.022822	1060	.000943396	.030715
1900	.000526316	.022942	1050	.000952381	.030861
1880	.000531915	.023063	1040	.000961538	.031009
1860	.000537634	.023187	1030	.000970873	.031159
1840	.000543478	.023313	1020	.000980392	.031311
1820	.000549450	.023440	1010	.000990099	.031466
1800	.000555555	.023570	1000	.001000000	.031623
1780	.000561798	.023702	999	.001001001	.031639
1760	.000568182	.023836	998	.001002004	.031654
1740	.000574712	.023973	997	.001003009	.031670
1720	.000581395	.024112	996	.001004016	.031686
1700	.000588235	.024254	995	.001005025	.031702
1680	.000595238	.024398	994	.001006036	.031718
1660	.000602409	.024744	993	.001007049	.031734
1640	.000609756	.024693	992	.001008065	.031750
1620	.000617284	.024845	991	.001009082	.031766
1600	.000625000	.025000	990	.001010101	.031782
1580	.000632911	.025158	989	.001011122	.031798
1560	.000641025	.025318	988	.001012146	.031814
1540	.000649351	.025482	987	.001013171	.031830
1520	.000657895	.025649	986	.001014199	.031847
1500	.000666666	.025820	985	.001015228	.031863
1480	.000675675	.025994	984	.001016260	.031879
1460	.000684932	.026171	983	.001017294	.031895
1440	.000694444	.026352	982	.001018330	.031911
1420	.000704225	.026537	981	.001019368	.031928
1400	.000714286	.026726	980	.001020408	.031944
1380	.000724638	.026919	979	.001021450	.031960
1360	.000735294	.027116	978	.001022495	.031977
1340	.000746268	.027318	977	.001023541	.031993
1320	.000757576	.027524	976	.001024590	.032009
1300	.000769231	.027735	975	.001025641	.032026
1280	.000781250	.027951	974	.001026694	.032042
1260	.000793651	.028172	973	.001027749	.032059
1240	.000806452	.028398	972	.001028807	.032075
1220	.000819672	.028630	971	.001029866	.032091
1200	.000833333	.028868	970	.001030928	.032108
1190	.000840336	.028988	969	.001031992	.032125
1180	.000847458	.029111	968	.001033058	.032141
1170	.000854701	.029235	967	.001034126	.032158
1160	.000862069	.029361	966	.001035197	.032174
1150	.000869566	.029488	965	.001036269	.032191
1140	.000877193	.029617	964	.001037344	.032208
1130	.000884956	.029748	963	.001038422	.032224
1120	.000892857	.029881	962	.001039501	.032241
1110	.000900900	.030015	961	.001040583	.032258
1100	.000909090	.030151	960	.001041667	.032275
1090	.000917431	.030289	959	.001042753	.032292

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=full of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
958	.001043841	.032309	908	.001101322	.033186
957	.001044932	.032325	907	.001102536	.033204
956	.001046025	.032342	906	.001103753	.033223
955	.001047120	.032359	905	.001104972	.033241
954	.001048218	.032376	904	.001106195	.033259
953	.001049318	.032393	903	.001107420	.033278
952	.001050420	.032410	902	.001108647	.033296
951	.001051525	.032427	901	.001109878	.033315
950	.001052632	.032444	900	.001111111	.033333
949	.001053741	.032461	899	.001112347	.033352
948	.001054852	.032479	898	.001113586	.033370
947	.001055966	.032496	897	.001114827	.033389
946	.001057082	.032513	896	.001116071	.033408
945	.001058201	.032530	895	.001117318	.033426
944	.001059322	.032547	894	.001118568	.033445
943	.001060445	.032565	893	.001119821	.033464
942	.001061571	.032582	892	.001121076	.033483
941	.001062699	.032599	891	.001122334	.033501
940	.001063830	.032616	890	.001123596	.033520
939	.001064963	.032634	889	.001124859	.033539
938	.001066098	.032651	888	.001126126	.033558
937	.001067236	.032669	887	.001127396	.033577
936	.001068376	.032686	886	.001128668	.033595
935	.001069519	.032703	885	.001129944	.033614
934	.001070664	.032721	884	.001131222	.033633
933	.001071811	.032738	883	.001132503	.033653
932	.001072961	.032756	882	.001133787	.033672
931	.001074114	.032774	881	.001135074	.033691
930	.001075269	.032791	880	.001136364	.033710
929	.001076426	.032809	879	.001137656	.033729
928	.001077586	.032826	878	.001138952	.033748
927	.001078749	.032844	877	.001140251	.033768
926	.001079914	.032862	876	.001141553	.033787
925	.001081081	.032879	875	.001142857	.033806
924	.001082251	.032897	874	.001144165	.033825
923	.001083423	.032915	873	.001145475	.033845
922	.001084599	.032933	872	.001146789	.033864
921	.001085776	.032951	871	.001148106	.033883
920	.001086957	.032969	870	.001149425	.033903
919	.001088139	.032987	869	.001150748	.033923
918	.001089325	.033005	868	.001152074	.033942
917	.001090513	.033023	867	.001153403	.033962
916	.001091703	.033041	866	.001154734	.033981
915	.001092896	.033059	865	.001156069	.034001
914	.001094092	.033077	864	.001157407	.034021
913	.001095290	.033095	863	.001158749	.034040
912	.001096491	.033113	862	.001160093	.034060
911	.001097695	.033131	861	.001161440	.034080
910	.001098901	.033149	860	.001162791	.034099
909	.001100110	.033168	859	.001164144	.034119

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
858	.001165501	.034139	808	.001237624	.035179
857	.001166861	.034159	807	.001239157	.035201
856	.001168224	.034179	806	.001240695	.035223
855	.001169591	.034199	805	.001242236	.035245
854	.001170960	.034219	804	.001243781	.035267
853	.001172333	.034239	803	.001245330	.035289
852	.001173709	.034259	802	.001246883	.035311
851	.001175088	.034279	801	.001248439	.035333
850	.001176471	.034300	800	.001250000	.035355
849	.001177856	.034320	799	.001251564	.035377
848	.001179245	.034340	798	.001253133	.035399
847	.001180638	.034360	797	.001254705	.035422
846	.001182033	.034381	796	.001256281	.035444
845	.001183432	.034401	795	.001257862	.035466
844	.001184834	.034421	794	.001259446	.035489
843	.001186240	.034442	793	.001261034	.035511
842	.001187648	.034462	792	.001262626	.035533
841	.001189061	.034483	791	.001264223	.035556
840	.001190476	.034503	790	.001265823	.035578
839	.001191895	.034524	789	.001267427	.035601
838	.001193317	.034544	788	.001269036	.035623
837	.001194743	.034565	787	.001270648	.035646
836	.001196172	.034586	786	.001272265	.035669
835	.001197605	.034606	785	.001273885	.035691
834	.001199041	.034627	784	.001275510	.035714
833	.001200480	.034648	783	.001277139	.035737
832	.001201923	.034669	782	.001278772	.035760
831	.001203369	.034689	781	.001280410	.035783
830	.001204819	.034710	780	.001282051	.035806
829	.001206273	.034731	779	.001283697	.035829
828	.001207729	.034752	778	.001285347	.035852
827	.001290190	.034773	777	.001287001	.035875
826	.001210654	.034794	776	.001288660	.035898
825	.001212121	.034816	775	.001290323	.035921
824	.001213592	.034837	774	.001291990	.035944
823	.001215067	.034858	773	.001293661	.035967
822	.001216545	.034879	772	.001295337	.035991
821	.001218027	.034900	771	.001297017	.036014
820	.001219512	.034922	770	.001298701	.036038
819	.001221001	.034943	769	.001300390	.036061
818	.001222494	.034964	768	.001302083	.036084
817	.001223990	.034985	767	.001303781	.036108
816	.001225490	.035007	766	.001305483	.036131
815	.001226994	.035028	765	.001307190	.036155
814	.001228501	.035050	764	.001308901	.036179
813	.001230012	.035071	763	.001310616	.036202
812	.001231527	.035093	762	.001312336	.036226
811	.001233046	.035115	761	.001314060	.036250
810	.001234568	.035136	760	.001315789	.036274
809	.001236094	.035158	759	.001317523	.036298

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
758	.001319261	.036322	708	.001412429	.037582
757	.001321004	.036346	707	.001414427	.037609
756	.001322751	.036370	706	.001416431	.037636
755	.001324503	.036394	705	.001418440	.037662
754	.001326260	.036418	704	.001420455	.037689
753	.001328021	.036442	703	.001422475	.037716
752	.001329787	.036466	702	.001424501	.037743
751	.001331558	.036490	701	.001426534	.037769
750	.001333333	.036515	700	.001428571	.037796
749	.001335113	.036539	699	.001430615	.037824
748	.001336898	.036563	698	.001432665	.037851
747	.001338688	.036588	697	.001434720	.037878
746	.001340483	.036613	696	.001436782	.037905
745	.001342282	.036637	695	.001438849	.037932
744	.001344086	.036662	694	.001440922	.037959
743	.001345895	.036686	693	.001443001	.037987
742	.001347709	.036711	692	.001445087	.038014
741	.001349528	.036736	691	.001447178	.038042
740	.001351351	.036761	690	.001449275	.038069
739	.001353180	.036786	689	.001451379	.038097
738	.001355014	.036810	688	.001453488	.038125
737	.001356852	.036835	687	.001455604	.038152
736	.001358696	.036860	686	.001457726	.038180
735	.001360544	.036885	685	.001459854	.038208
734	.001362398	.036911	684	.001461988	.038236
733	.001364256	.036936	683	.001464129	.038264
732	.001366120	.036961	682	.001466276	.038292
731	.001367989	.036986	681	.001468429	.038320
730	.001369863	.037012	680	.001470588	.038348
729	.001371742	.037037	679	.001472754	.038376
728	.001373626	.037063	678	.001474926	.038405
727	.001375516	.037088	677	.001477105	.038433
726	.001377410	.037113	676	.001479290	.038461
725	.001379310	.037139	675	.001481481	.038490
724	.001381215	.037164	674	.001483680	.038518
723	.001383126	.037190	673	.001485884	.038547
722	.001385042	.037216	672	.001488095	.038576
721	.001386963	.037242	671	.001490313	.038604
720	.001388889	.037268	670	.001492537	.038633
719	.001390821	.037294	669	.001494768	.038662
718	.001392758	.037320	668	.001497006	.038691
717	.001394700	.037346	667	.001499250	.038720
716	.001396648	.037372	666	.001501502	.038749
715	.001398601	.037398	665	.001503759	.038778
714	.001400560	.037424	664	.001506024	.038808
713	.001402525	.037450	663	.001508296	.038837
712	.001404494	.037477	662	.001510574	.038866
711	.001406470	.037503	661	.001512859	.038895
710	.001408451	.037529	660	.001515152	.038925
709	.001410437	.037556	659	.001517451	.038954

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any diameter (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
658	.001519757	.038984	608	.001644737	.040555
657	.001522070	.039013	607	.001647446	.040589
656	.001524390	.039043	606	.001650165	.040622
655	.001526718	.039073	605	.001652893	.040656
654	.001529052	.039103	604	.001655629	.040689
653	.001531394	.038133	603	.001658375	.040723
652	.001533742	.039163	602	.001661130	.040757
651	001536098	.039193	601	.001663894	.040791
650	.001538462	.039223	600	.001666667	.040825
649	.001540832	.039253	599	.001669449	.040859
648	.001543210	.039284	598	.001672241	.040893
647	.001545595	.039314	597	.001675042	.040927
646	.001547988	.039344	596	.001677852	.040961
645	.001550388	.039375	595	.001680672	.040996
644	.001552795	039405	594	.001683502	.041031
643	.001555210	.039436	593	.001686341	.041065
642	.001557632	.039467	592	.001689189	.041100
641	.001560062	.039498	591	.001692047	.041135
640	.001562500	.039528	590	.001694915	.041169
639	.001564945	.039559	589	.001697793	.041204
638	.001567398	.039590	588	.001700680	.041239
637	.001569859	.039621	587	.001703578	.041274
636	.001572327	.039653	586	.001706485	.041309
635	.001574803	.039684	585	.001709420	.041345
634	.001577287	.039715	584	.001712329	.041380
633	.001579779	.039746	583	.001715266	.041416
632	.001582278	.039778	582	.001718213	.041451
631	.001584786	.039809	581	.001721170	.041487
630	.001587302	.039841	580	.001724138	.041523
629	.001589825	.039873	579	.001727116	.041559
628	.001592357	.039904	578	.001730104	.041594
627	.001594896	.039936	577	.001733102	.041630
626	.001597444	.039968	576	.001736111	.041667
625	.001600000	.040000	575	.001739130	.041703
624	.001602564	.040032	574	.001742160	.041739
623	.001605136	.040064	573	.001745201	.041776
622	.001607717	.040096	572	.001748252	.041812
621	.001610306	.040128	571	.001751313	.041848
620	.001612903	.040161	570	.001754386	.041885
619	.001615509	.040193	569	.001757469	.041922
618	.001618123	.040226	568	.001760563	.041959
617	.001620746	.040258	567	.001763668	.041996
616	.001623377	.040291	566	.001766784	.042033
615	.001626016	.040324	565	.001769912	.042070
614	.001628664	.040357	564	.001773050	.042108
613	.001631321	.040389	563	.001776199	.042145
612	.001633987	.040422	562	.001779359	.042183
611	.001636661	.040456	561	.001782531	.042220
610	.001639344	.040489	560	.001785714	.042258
609	.001642036	.040522	559	.001788909	.042295

TABLE 3.—GIVING VALUES OF a AND r AND ALSO THE FACTORS $c\sqrt{r}$ AND $ac\sqrt{r}$ FOR CORRESPONDING TRANSVERSE DIAMETERS OF EGG-SHAPED SEWERS FLOWING *full depth* GIVEN IN FIRST COLUMN.

These factors are to be used only where the value of n, that is the coefficient of roughness of lining of channel = .015, as in second-class or rough-faced brickwork, well dressed stone work, foul and slightly tuberculated iron, cement and terra-cotta pipes with imperfect joints and in bad order.

Area of egg-shaped sewer flowing full depth = $D^2 \times 1.148525$.
Perimeter of egg-shaped sewer flowing full depth = $D \times 3.9649$
Hydraulic mean depth of egg-shaped sewer flowing full depth = $D \times 0.2897$.

$$v = c\sqrt{r} \times \sqrt{s}. \quad Q = av = ac\sqrt{r} \times \sqrt{s}.$$

D= transverse diam. ft.	in.	a = area in square ft.	r = hydraulic mean depth in feet.	For velocity. $c\sqrt{r}$	For discharge $ac\sqrt{r}$	D= transverse diam. ft.	in.	a = area in square ft.	r = hydraulic mean depth in feet.	For velocity. $c\sqrt{r}$	For discharge $ac\sqrt{r}$
1	0	1.148	.2897	39.62	45.528	5	2	30.660	1.497	130.7	4007.9
1	2	1.563	.3380	44.66	69.804	5	4	32.669	1.545	133.6	4364.9
1	4	2.041	.3864	49.57	101.17	5	6	34.743	1.593	136.4	4738.0
1	6	2.584	.4345	54.08	139.74	5	8	36.880	1.642	139.2	5131.7
1	8	3.190	.4828	58.64	187.06	5	10	39.081	1.690	142.0	5548.0
1	10	3.860	.5311	62.83	242.52	6	0	41.347	1.738	144.6	5980.3
2	0	4.594	.5794	66.93	307.48	6	2	43.676	1.787	147.3	6435.1
2	2	5.391	.6277	71.01	382.81	6	4	46.068	1.835	149.8	6902.6
2	4	6.253	.6760	74.93	468.54	6	6	48.525	1 883	152.5	7399.3
2	6	7.178	.7242	78.76	565.34	6	8	51.046	1.931	155.2	7920.6
2	8	8.167	.7725	82.44	673.29	6	10	53.629	1.980	157.7	8547.1
2	10	9.220	.8208	86.21	794.86	7	0	56.278	2.028	160.2	9015.7
3	0	10.337	.8691	89.70	927.23	7	4	61.764	2.124	165.0	10192
3	2	11.517	.9174	93.25	1074.0	7	8	67.508	2.221	170.1	11482
3	4	12.761	.9657	96.73	1234.4	8	0	73.506	2.318	174.8	12852
3	6	14.069	1.014	100.1	1407.6	8	4	79.758	2.414	179.6	14327
3	8	15.442	1.062	103.4	1596.7	8	8	86.268	2.511	184.3	15898
3	10	16.877	1.111	106.6	1799.1	9	0	93.030	2.607	188.8	17563
4	0	18.376	1.159	109.9	2019.5	9	4	100.049	2.704	193.1	19323
4	2	19.940	1.207	113.0	2254.0	9	8	107.324	2.800	197.5	21198
4	4	21.566	1.255	116.0	2501.4	10	0	114.853	2.897	201.9	23191
4	6	23.258	1.304	119.1	2770.0	10	6	126.625	3.042	208.3	26376
4	8	25.013	1.352	122.1	3053.8	11	0	138.972	3.187	214.6	29822
4	10	26.830	1.400	125.0	3353.0	12	0	165.388	3.476	226.8	37502
5	0	28.713	1.449	128.0	3675.6						

TABLE 4.—GIVING VALUES OF a AND r AND ALSO THE FACTORS $c\sqrt{r}$ AND $ac\sqrt{r}$ FOR CORRESPONDING DIAMETERS OF EGG SHAPED SEWERS FLOWING *two-thirds full depth* GIVEN IN FIRST COLUMN.

These factors are to be used only where the value of n, that is the coefficient of roughnes of lining of channel = .015 as in second class or rough-faced brickwork, well-dressed stone work, foul and slightly tuberculated iron, cement and terra-cotta pipes with imperfect joints and in bad order.

Area of section of egg-shaped sewer flowing two-thirds full depth = $D^2 \times 0.755825$.
Perimeter of section of egg-shaped sewer flowing two-thirds full depth = $D \times 2.3941$.
Hydraulic mean depth of section of egg-shaped sewer flowing two-thirds full depth = $D \times 0.3157$.

$$v = c\sqrt{r} \times \sqrt{s}. \quad Q = av = ac\sqrt{r} \times \sqrt{s}.$$

D = transverse diam. ft.	in.	a = area in square ft.	r = hydraulic mean depth in feet.	For velocity. $c\sqrt{r}$	For discharge. $ac\sqrt{r}$	D = transverse diam. ft.	in.	a = area in square ft.	r = hydraulic mean depth in feet.	For velocity. $c\sqrt{r}$	For discharge $ac\sqrt{r}$
1	0	0.756	0.316	42.40	32.048	5	2	20.177	1.631	138.6	2795.9
1	2	1.029	0.368	47.80	49.181	5	4	21.498	1.684	141.7	3045.5
1	4	1.344	0.421	52.82	70.993	5	6	22.863	1.736	144.6	3305.3
1	6	1.701	0.474	57.68	98.115	5	8	24.270	1.789	147.5	3578.9
1	8	2.099	0.526	62.46	131.10	5	10	25.718	1.842	150.3	3864.8
1	10	2.540	0.579	66.94	170.02	6	0	27.210	1.894	153.1	4165.3
2	0	3.023	0.631	71.42	216.54	6	2	28.743	1.947	155.9	4481.6
2	2	3.548	0.684	75.59	268.19	6	4	30.317	1.999	158.7	4811.9
2	4	4.115	0.737	79.69	327.93	6	6	31.933	2.052	161.5	5158.5
2	6	4.724	0.789	83.90	396.32	6	8	33.592	2.095	164.2	5516.6
2	8	5.375	0.842	87.82	472.01	6	10	35.292	2.157	166.9	5891.0
2	10	6.067	0.894	91.60	555.74	7	0	37.035	2.210	169.6	6283.5
3	0	6.802	0.947	95.33	648.40	7	4	40.646	2.315	174.8	7106.8
3	2	7.579	1.000	99.10	751.08	7	8	44.426	2.420	179.9	7993.0
3	4	8.398	1.052	102.7	862.41	8	0	48.373	2.526	184.9	8944.0
3	6	9.259	1.105	106.2	983.24	8	4	52.487	2.631	189.8	9964.1
3	8	10.161	1.158	109.7	1115.1	8	8	56.771	2.736	194.6	11050
3	10	11.106	1.210	113.2	1256.1	9	0	61.222	2.841	199 5	12213
4	0	12.093	1.263	116.5	1409.4	9	4	65.840	2.947	204.2	13444
4	2	13.123	1.315	119.8	1572.1	9	8	70.628	3.052	208.7	14743
4	4	14.192	1.368	123.1	1746.9	10	0	75.583	3.157	213.3	16125
4	6	15.305	1.421	126.3	1932.7	10	6	83.330	3.315	220.1	18342
4	8	16.460	1.473	129.4	2130.5	11	0	91.455	3.473	226.8	20738
4	10	17.656	1.526	132.5	2338.6	12	0	108.839	3.788	239.4	26060
5	0	18.895	1.579	135.5	2560.3						

TABLE 5.—GIVING VALUES OF a AND r AND ALSO THE FACTORS $c\sqrt{r}$ AND $ac\sqrt{r}$ FOR CORRESPONDING DIAMETERS OF EGG-SHAPED SEWERS FLOWING *one-third full depth* GIVEN IN FIRST COLUMN.

These factors are to be used only where the value of n, that is the coefficient of roughness of lining of channel = 015 as in second-class or rough-faced brickwork, well dressed stone work, foul and slightly tuberculated iron, cement and terra-cotta pipes with imperfect joints and in bad order.

Area of section of egg-shaped sewers flowing one-third full depth = $D^2 \times 0.284$.
Perimeter of section of egg-shaped sewer flowing one-third full depth = $D \times 1.3747$.
Hydraulic mean depth of section of egg-shaped sewers flowing one-third full depth = $D \times 0.2066$

$$v = c\sqrt{r} \times \sqrt{s}. \quad Q = av = ac\sqrt{r} \times \sqrt{s}.$$

D = transverse diam. ft.	in.	a = area in square ft.	r = hydraulic mean depth in feet.	For velocity. $c\sqrt{r}$	For discharge $ac\sqrt{r}$	D = transverse diam. ft.	in.	a = area in square ft.	r = hydraulic mean depth in feet.	For velocity. $c\sqrt{r}$	For discharge $ac\sqrt{r}$
1	0	0.284	0.207	30.41	8.637	5	2	7.581	1.068	103.7	785.86
1	2	0.387	0.241	34.38	13.303	5	4	8.078	1.102	106.1	856.67
1	4	0.505	0.276	38.16	19.269	5	6	8.591	1.136	108.3	930.54
1	6	0.639	0.310	42.23	26.980	5	8	9.120	1.171	110.6	1008.7
1	8	0.789	0.344	45.39	35.815	5	10	9.664	1.205	112.9	1091.0
1	10	0.955	0.379	48.74	46.546	6	0	10.224	1.240	115.0	1175.8
2	0	1.136	0 413	52.09	59.173	6	2	10.800	1.274	117.3	1266.4
2	2	1.333	0 448	55.29	73.696	6	4	11.391	1.309	119.4	1359.8
2	4	1.546	0.482	58.58	90.568	6	6	12.999	1.343	121.5	1458.1
2	6	1.776	0.517	61.58	109.37	6	8	12.622	1.377	123.7	1561.0
2	8	2.020	0.551	64.49	130.26	6	10	13.261	1.412	125.8	1668.8
2	10	2.280	0.585	67.46	153.80	7	0	13.916	1.446	127.9	1779.4
3	0	2.556	0.620	70.48	180.14	7	4	15.273	1.515	131.9	2014.1
3	2	2.848	0.654	73.24	208.98	7	8	16.693	1.584	135.8	2266.7
3	4	2.156	0.689	75.98	239.79	8	0	18.176	1.653	139.9	2542.7
3	6	3.479	0.723	78.63	273.54	8	4	19.722	1.722	143.7	2833.8
3	8	3.818	0.758	81.31	310.44	8	8	21.332	1.791	147.5	3146.2
3	10	4.173	0.792	84.03	350.67	9	0	23.004	1.859	151.3	3480.7
4	0	4.544	0.826	86.61	393.55	9	4	24.739	1.928	155.0	3834.7
4	2	4.931	0.861	88.98	438.75	9	8	26.538	1.997	158.6	4208.4
4	4	5.333	0.895	91.60	488.50	10	0	28.400	2.066	162.1	4604.7
4	6	5.751	0.930	94.08	541.04	10	6	31.311	2.169	167.5	5245.3
4	8	6.185	0.964	96.57	597.29	11	0	34.364	2.273	172.6	5932.1
4	10	6.635	0.999	99.10	657.53	12	0	40.892	2.479	183.1	7489.0
5	0	7.100	1.033	101.3	719.27						

DYNAMO-ELECTRIC MACHINERY.

By PROFESSOR SILVANUS P. THOMPSON, B. A., D. Sc., M. S. T. E.

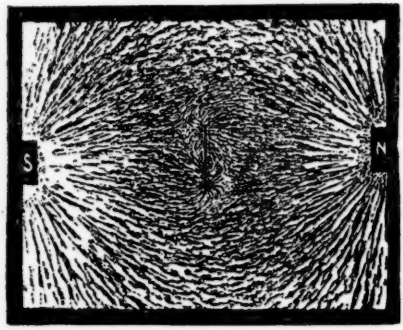
From the "Journal of the Society of Arts."

III.

The Dynamo as a Motor.

In my first lecture, I laid down the definition that dynamo-electric machinery meant "machinery for converting the energy of mechanical motion into the energy of electric currents, or *vice versa*." In the two lectures which I have already had the honor of delivering, I have treated the dynamo solely in its functions as a generator of electric currents. In this third lecture I come to the converse function of the dynamo, namely, that of converting the energy of electric currents into the energy of mechanical motion.

Fig. 46.

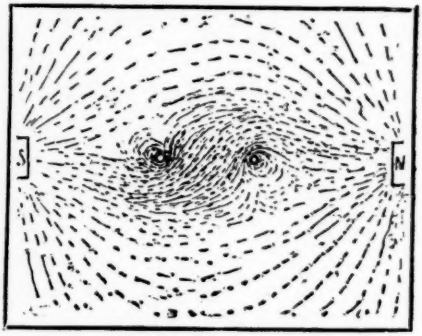


ACTION OF MAGNETIC FIELD ON A MAGNETIC NEEDLE.

An electric-motor, or, as it was formerly called, an electro magnet engine, is one which does mechanical work at the expense of electric energy; and this is true, no matter whether the magnets which form the fixed part of the machine be permanent magnets of steel or electro-magnets. Any one, in fact, of the four kinds of dynamo, can be used conversely as a motor, though, as we shall see, some more appropriately than others. But whether their field magnets be of permanently magnetized steel or of temporarily magnetized iron, all these motors are electro magnetic in principle; that is to say, there is some part either fixed or moving which is an electro-magnet, and which as such attracts and is attracted magnetically.

Every one knows that a magnet will attract the opposite pole of another magnet, and will pull it round. We know, also, that every magnet placed in a magnetic field tends to turn round and set itself along the lines of force. Let me, as a first illustration of this class of actions, exhibit to you the nature of the forces at work in the magnetic field. In the figure

Fig. 47.



ACTION OF MAGNETIC FIELD ON A WIRE CARRYING A CURRENT.

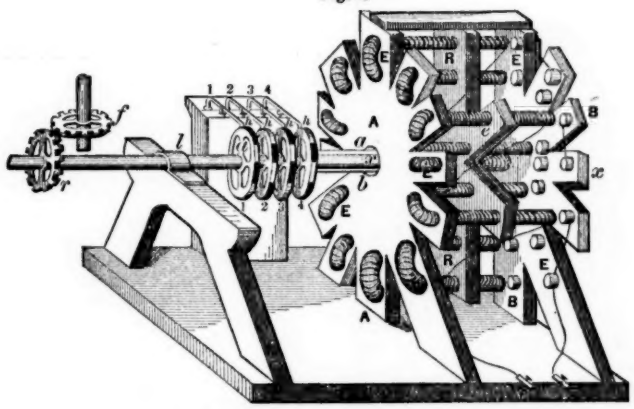
which I now throw upon the screen (Fig. 46), we have, in the first place, a simple magnetic field produced between the poles of two strong magnets, one on the right, the other on the left. Between the two, confined forcibly at right-angles to the lines of force, I hold a small magnetic needle. Iron filings sprinkled in the field reveal the actions at work in a most instructive way. Faraday, who first taught us the significance of these mysterious lines of force, has told us that we may reason about them as if they tended to contract or grow shorter. Now a simple

inspection of Fig. 46 will show that the shortening of the lines of force must have the effect of rotating the magnetic needle upon its center, through an angle of 90°, for the lines stream away on the right hand above, and on the left hand below, in a most suggestive fashion. It is not, therefore, difficult to understand that very soon after the invention of the electro-magnet, which gave us for the first time a magnet whose power was under control, a number of ingenious persons perceived that it would .be possible to construct an electro-magnetic engine in which an electro-magnet, placed in a magnetic field, should be pulled round; and further, that the rotation should be kept up continuously, by reversing the current at an appropriate moment. As a matter of fact, a mere coil of wire, carrying a current, is acted upon when placed in the magnetic field, and is pulled round as a magnet is. Fig. 47 shows how, in this case, the lines of force reveal the action. The magnetic field is, as before produced, between the ends of two large magnets. The two round spots are two holes drilled in the sheet of glass, where the wire which carried the current came up through the glass and descended again. You will notice how the lines of iron filings, which would, if there were no current, run simply across from left to right, are bent out of their course. If these lines could shorten themselves, they must, of necessity, twist the loop of wire round, and cause it to set at right angles to its present position.

On this very principle was constructed the earliest electric motor of Ritchie, so well known in many forms as a stock piece of electric apparatus, but little better in reality than a toy.

A great step in advance was made by Jacobi, who, in 1838, constructed the multipolar machine, of which we give a representation in Fig. 48. This motor, which Jacobi designed for his electric boat, had two strong wooden frames, A and B, in each of which a dozen electro-magnets were fixed, their poles being set alternately. Between them, upon a wood-

Fig.48



JACOBI'S ELECTRIC MOTOR (1838.)

en disk, were placed another set of electro-magnets, which, by the alternate attraction and repulsion of the fixed poles, were kept in rotation, the current which traversed the rotating magnets being regularly reversed at the moment of passing the poles of the fixed magnets by means of a commutator, consisting, according to Jacobi's directions, of four brass-toothed wheels, having pieces of ivory or wood let in between the teeth for insulation. Jacobi's motor is, in fact, a very advanced type of dynamo, and differs very little in point of design from one of Wilde's most successful forms.*

* Wilde's is, however, designed as a generator. Jacobi's, on the contrary, was designed as a motor; though, of course it would generate currents if driven round by mechanical power.

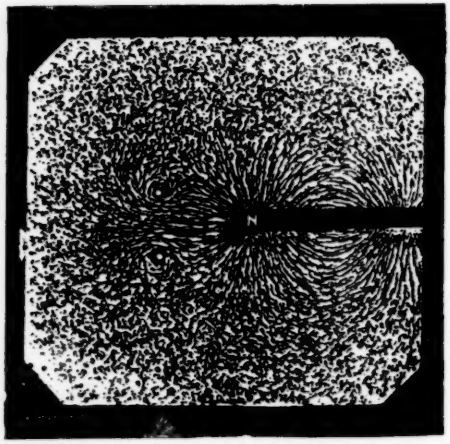
A still earlier rotating apparatus, and, like Ritchie's motor, a mere toy, was Barlow's wheel, Fig. 43, described in 1823. This instrument, interesting as being the forerunner of Faraday's disk dynamo, is the representative of an important class of machines, namely, those which have a sliding contact merely, and need no commutator.

A fourth class of motors may be named, wherein the moving part, instead of rotating upon an axis, is caused to oscillate backwards and forwards. Professor Henry, to who we owe so much in the early history of electro-magnetism, constructed, in 1831, a motor with an oscillating beam, alternately drawn backwards and forwards by the intermittent action of an electro magnet. Dal Negro's motor of 1833 was of this class; in it a steel rod was caused to oscillate between the poles of an electro-magnet, and caused a crank, to which it was geared, to rotate in consequence. A distinct improvement in this type of machine was introduced by Page, who employed hollow coils or bobbins as electro-magnets, which, by their alternate action, sucked down iron cores into the coils, and caused them to oscillate to and fro. Motors of this kind form an admirable illustration of one of the laws of electro-magnetics, first formulated by Gauss, but developed later by Maxwell, to the effect that a circuit acts on a magnetic pole in such a way as to make the number of magnetic lines of force that pass through the circuit a maximum. Once more I have recourse to iron filings to illustrate this abstract proposition of electric geometry.

In the figure before you (Fig. 49), the north pole of a bar magnet is placed opposite a circuit or loop of wire traversed by a current, and which comes up through the glass at the lower hole, and descends at the upper hole. The tendency to draw as many as possible of the magnet's lines of force into the embrace of the circuit is unmistakable. If now we reverse the current, what do we find? Fig. 50 supplies the answer; for now we find that the magnet's lines of force, instead of being drawn in, are pushed out. In fact, in one case, the pole is attracted, in the other repelled.

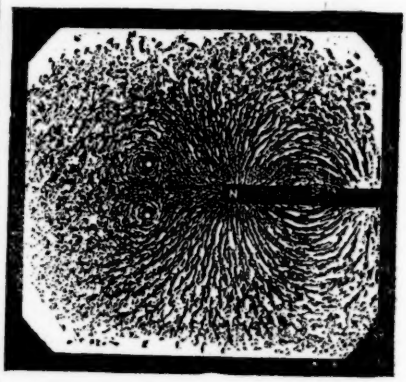
Page's suggestion was further developed by Bourbouze, who constructed the curious motor which looks uncommonly like an old type of steam-engine. We have here a beam, a crank, fly-wheel, connecting-rod, and even an eccentric valve-gear and a slide valve. But for cylinders we have four hollow electro-magnets; for pistons we have iron cores, that are

Fig. 49.



POLE OF MAGNET ATTRACTED INTO A CIRCUIT TRAVERSED BY A CURRENT.

Fig. 50.



POLE OF MAGNET REPELLED OUT OF THE CIRCUIT WHEN THE CURRENT IS REVERSED.

alternately sucked in and repelled out; and for slide valve we have a commutator, which, by dragging a pair of platinum-tipped springs over a flat surface made of three pieces of brass separated by two insulating strips of ivory, reverses at every stroke the direction of the currents in the coils of the electro-magnets. It is really a very ingenious machine, but, in point of efficiency, far behind many other electric motors. Unfortunately, it does not do to design dynamo-electric machinery on the same lines as steam-engines.

Yet, a fifth class of electric-motors owes its existence to Froment, who, fixing a series of parallel iron bars upon the periphery of a drum, caused them to be attracted, one after the other, by an electro-magnet or electro-magnets, and thus procured a continuous rotation.

Lastly, of the various types of motors we may enumerate a class in which the rotating portion is enclosed in an eccentric frame of iron, so that as it rotates, it gradually approaches nearer. Little motors, working on this principle of "oblique approach," were invented by Wheatstone, and have long been used for spinning Geissler tubes, and other light experimental work. More recently, Trouvé and Weisendanger have sought to embody this principle in motors of more ambitious proportions, but without securing any great advantage.

It would be impossible, within the limits of a lecture, to deal with a tithe of all the various stages of discovery and invention, and if it were my intention to deal with the subject from the historical point of view, I might speak of many interesting and curious machines that have from time to time been tried. I might tell you how Page, after inventing his machine in 1834, succeeded, in 1852, in constructing a motor of such a size that he was able to drive a circular-saw and a lathe by it. I might describe the electric-motor of Davidson, which, in 1842, enabled him to propel a carriage at the speed of four miles an hour, between Edinburgh and Glasgow. I might describe the engine built in 1849, by Soren Hjörth, at Liverpool, which was of ten-horse power.

All these early attempts, however, came to nothing, for two reasons. At that time there was no economical method of generating electric currents known. At that time, moreover, the great physical law of the conservation of energy was not recognized, and its all-important bearings upon the theory of electric machinery were not available.

While voltaic batteries were the only available sources of electric currents, economical working of electric-motors was hopeless. For a voltaic battery, wherein electric currents are generated by dissolving zinc in sulphuric acid, is a very expensive source of power. To say nothing of the cost of the acid, the zinc—the very fuel of the battery—costs more than twenty times as much as coal, and is a far worse fuel; for whilst an ounce of zinc will evolve heat to an amount equivalent to 113,000 foot-pounds of work, an ounce of coal will furnish the equivalent of 695,000 foot-pounds.

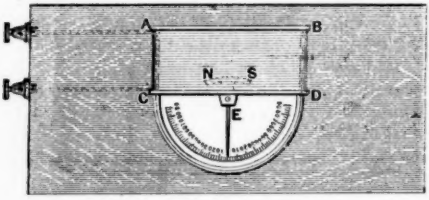
The fact, however, which seemed most discouraging, and which, if rightly interpreted in accordance with the law of conservation of energy, would have been found to be (on the contrary) a most encouraging fact, was the following:—If a galvanometer was placed in the circuit with the electric-motor and the battery, it was found that when the motor was running it was impossible to force so strong a current through the wires as that which flowed when the motor was standing still. Now there are only two causes that can stop such a current flowing in a circuit; there must be either an obstructive resistance or else a counter-electromotive force. At first, the common idea was that, when the motor was spinning round, it offered a greater resistance to the passage of the electric current than when it stood still. The genius of Jacobi enabled him, however, to discern that the observed diminution of current was really due to the fact that the motor, by the act of spinning round, began to work as a dynamo on its own account, and tended to set up a current in the circuit in the opposite direction to that which was driving it. The faster it rotated the greater was the counter-electromotive force (or "electromotive force of reaction") which was developed. In fact, the theory of conservation of energy requires that such a reaction should exist.

We know that, in the converse case, when we are employing mechanical power to generate currents by rotating a dynamo, directly we begin to generate cur-

rents, that is to say, directly we begin to do electric *work*, it immediately becomes much harder work to turn the dynamo than is the case when no electric work is being done. In other words, there is an opposing reaction to the mechanical force which we apply in order to do electric work. An opposing reaction to a mechanical force may be termed a "counter-force." When, on the other hand, we apply (by means of a voltaic battery, for example) an electromotive force to do mechanical work, we find that here again there is an opposing reaction; and an opposing reaction to an electromotive force is a "counter-electromotive force."

The experiment of showing the existence of this counter-electromotive force is a very easy one. All one requires is a

Fig. 51



S. P. THOMPSON'S LANTERN GALVANOMETER.

little motor, a few cells of battery, and a galvanometer. The galvanometer I shall use to-night is the one which I brought out some years ago, and which has proved itself very convenient for lecture work, because it can be put into any ordinary lantern and projected on the screen (Fig. 51). I have here, as a battery, four small accumulators of the Faure-Sellon-Volckmar type, and I have connected them with a little motor, also of my own design, the current being arranged so as to run through the galvanometer. I hold the spindle of the motor fast, so that it cannot rotate, and you see that the pointer of the galvanometer indicates 44°. I now release the motor; it begins to rotate, and as its speed increases, you observe the needle descends the scale to 23°, and eventually to about 15°. If I load the motor and cause it to slacken speed, the needle at once returns.

The existence of this counter-electromotive force is of the utmost importance, in considering the action of the dynamo as a motor, because upon the existence and magnitude of this counter-electromotive force, depends the degree to which any given motor enables us to utilize electric energy that is supplied to it in the form of an electric current. In discussing the dynamo as a generator, I pointed out many considerations, the observance of which would tend to improve the efficiency of such generators. It is needless to say that many of these considerations, such as the avoidance of useless resistances, unnecessary iron masses in cores and the like, will also apply to motors. The freer a motor is from such objections, the more efficient will it be. But the efficiency of a motor in utilizing the energy of a current depends not only on its efficiency in itself, but on another consideration, namely, the relation between the electromotive force which it itself generates when rotating, and the electromotive force—or, as some people call it, the electric pressure—at which the current is supplied to it. A motor which itself in running generates only a *low* electromotive force cannot, however well designed, be an *efficient* or economical motor when supplied with currents at a *high* electromotive force. A good low-pressure steam-engine does not become more "efficient" by being supplied with high-pressure steam. Nor can a high-pressure steam-engine, however well constructed, attain a high efficiency when worked with steam at low pressures. Analogous considerations apply to dynamos used as motors. They must be supplied with currents at electromotive forces adapted to them. Even a perfect motor —one without friction or resistance of any kind—cannot give an "efficient" or economical result, if the law of efficiency is not observed in the conditions under which the electric current is supplied to it.

Now it can be shown, mathematically, that the efficiency with which a perfect motor utilizes the electric energy of the current depends upon the ratio between this counter-electromotive force and the electromotive force of the current that is supplied by the battery. No motor ever succeeds in turning into useful work the whole of the currents that feed it, for it is impossible to construct machines without resistance, and whenever resistance is offered to a current part of the energy

of the current, it is wasted in heating the resisting wire. Let the symbol W stand for the whole electric energy of a current, and let w stand for that part of the energy which the motor takes up as useful work from the circuit.* All the rest of the energy of the current, or W—w, will be wasted in useless heating of the resistances.

But if we want to work our motor under the conditions of greatest economy, it is clear that we must have as little heat-waste as possible; or, in symbols, w must be as nearly as possible equal to W. It can be shown, mathematically, that the ratio between the useful energy thus appropriated, and the total energy spent, is equal to the ratio between the counter-electromotive force of the motor, and the whole electromotive force of the battery that feeds the motor. The proof will be given later. Let us call this whole electromotive force with which the battery feeds the motor E, and let us call the counter-electromotive force e. Then the rule is

$$w : W = e : E$$

or, if we express the efficiency as a fraction,

$$\frac{w}{W} = \frac{e}{E}.$$

But we may go one stage further. If the resistances of the circuit are constant, the current c, observed when the motor is running, will be less than C, the current while the motor was standing still. But from Ohm's law, we know that

$$c = \frac{E - e}{R},$$

hence

$$\frac{C - c}{C} = \frac{e}{E} = \frac{w}{W}.$$

From which it appears that we can calculate the efficiency at which the motor is working, by observing the ratio between the fall in the strength of the current and the original strength. Now, this mathematical law of efficiency has been known for twenty years,* but has been strangely misapprehended. Another law, discovered by Jacobi, not a law of efficiency at all, but a law of maximum work in a given time, has usually been given instead.

Jacobi's law concerning the maximum work of an electric-motor supplied with currents from a source of given electromotive force, is the following:—The mechanical work given out by a motor is a maximum when the motor is geared to run at such a speed that the current is reduced to half the strength that it would have if the motor was stopped. This, of course, implies that the counter-electromotive force of the motor is equal to half the electromotive force furnished by the battery or generator. Now, under these circumstances, only half the energy furnished by the external source is utilized, the other half being wasted in heating the circuit. If Jacobi's law were indeed the law of efficiency, no motor, however perfect in itself, could convert more than 50 per cent. of the electric energy supplied to it into actual work. Now Siemens showed,† some years ago, that a dynamo can be, in practice so used as to give out more than 50 per cent. of the energy of the current. It can, in fact, work more efficiently if it be not expected to do its work so quickly. Dr. Siemens, to whom we owe the honor of having first shown us the true physical signification of the mathematical expressions which, until then, had been regarded as mere abstractions, has, in fact, proved that if the motor be arranged so as to do its work at less than the maximum rate, by being geared so as to do much less work per revolution, but yet so as to run at a higher speed, it will be more efficient; that is to say, though it does less work, there will also be still less electric energy expended, and the ratio of the useful work done to the energy expended will be nearer unity than before.

The algebraic reasoning is as follows: If E be the electromotive force of the generator when the motor is at rest, and

* This symbol w must be clearly understood to refer to the value of the work taken up by the motor, as measured electrically. The whole of this work will not appear as useful mechanical effect, however, for part will be lost by mechanical friction, and part also in the wasteful production of eddy-currents in the moving parts of the motor. What proportion of w appears as useful mechanical work depends on the efficiency of the motor *per se*, which we are not here considering. In all that follows immedi.tely we shall suppose such causes of loss not to exist; or the motor will be considered as a perfect motor.

* See Verdet's "Théorie Mécanique de la Chaleur," where, however, Verdet makes the very mistake so often made, of supposing that the greatest possible efficiency of a motor, working with a given electromotive force, is 50 per cent, or its efficiency when working at the maximum rate.

† The matter was also very well and clearly put by Prof. W. E. Ayrton, in his lecture on "Electric Transmission of Power," before the British Association in Sheffield, in 1879.

c be the current which flows at any time, the electric energy W, expended in unit time, will be (as expressed in Watts) given by the equation

$$W = Ec = E\frac{(E-e)}{R} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now, when the motor is running, part of this electric energy is being spent in doing work, and the remainder is wasting itself in heating the wires of the circuit. We have already used the symbol w for the useful work (per second) done by the motor. All the energy which is not thus utilized is wasted in heating the resistances. Let the symbol H represent this heat. Its mechanical value will be HJ, where J stands for Joule's equivalent. Then clearly we shall have

$$W = w + HJ.$$

But, by Joule's law, the heat-waste of the current, whose strength is c running through resistance R, is expressed by the equation

$$HJ = c^2R.$$

Substituting this value above, we get

$$W = w + c^2R \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which we may also write

$$w = W - c^2R.$$

But by equation (1) $W = Ec$, whence

$$w = Ec - c^2R \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and, writing for c its value, $\frac{E-e}{R}$, we get

$$w = \frac{(E-e)\,(E-[E-e])}{R}$$

or

$$w = e\frac{E-e}{R} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Comparing equation (5) with equation (1), we get the following:

$$\frac{w}{W} = \frac{e\,(E-e)}{E(E-e)}.$$

or, finally,

$$\frac{w}{W} = \frac{e}{E}.$$

This is, in fact, the mathematical law of efficiency, so long misunderstood until Siemens showed its significance. We may appropriately call it the law of Siemens. Here the ratio $\frac{w}{W}$ is the measure of the efficiency of the motor, and the equation shows that we may make this efficiency as nearly equal to unity as we please, by letting the motor run so fast that e is very nearly equal to E: which is the true law of efficiency of a perfect motor supplied with electric energy, under the condition of constant external electromotive force.

Now go back to equation (3), which is:

$$w = Ec - c^2R.$$

In order to find what value of c will give us the maximum value for w (which is the work done by the motor in unit time) we must take the differential coefficient and equate it to zero.

$$\frac{dw}{dc} = E - 2cR = o,$$

whence we have

$$c = \tfrac{1}{2}\frac{E}{R}.$$

But, by Ohm's law, $\frac{E}{R}$ is the value of the current when the motor stands still. So we see at once that, to get maximum work per second out of our motor, the motor must run at such a speed as to bring down the current to half the value which it would have if the motor were at rest. In fact, we here prove the law of Jacobi for the maximum rate of doing work. But here since

$$c = \frac{E-e}{R} = \tfrac{1}{2}\frac{E}{R},$$

it follows that

$$E - e = \tfrac{1}{2}E;$$

or

$$\frac{e}{E} = \tfrac{1}{2},$$

whence it follows also that

$$\frac{W}{w} = \tfrac{1}{2}.$$

That is to say, the efficiency is but 50 per cent. when the motor does its work at the maximum rate.*

* It may be worth while to recall a precisely parallel case that occurs in calculating the currents from a voltaic battery. Every one is familiar with the rule for grouping a battery which consists of a given number of cells, that they will yield a maximum current through a given external resistance when so grouped that the internal resistance of the battery shall, as nearly as possible, equal the external resistance. But this rule, which is true for maximum current (and, therefore, for maximum rate of using up the zincs of one's battery) is not the case of greatest economy. For if external and internal resistance are equal, half the energy of the current will be wasted in heat in the cells, and half only will be available in the external

Now, though several graphic constructions have been suggested to convey these facts to the eye, none have hitherto been, to my mind, quite satisfactory. I have, therefore, worked out a new construction, which enables me, in one diagram, to exhibit graphically both Jacobi's law of maximum rate of working, and Siemen's law of efficiency.

Let the vertical line, A B (Fig. 52) represent the electromotive force E, of the electric supply. On A B construct a square, A B C D, of which let the diagonal

Fig.52

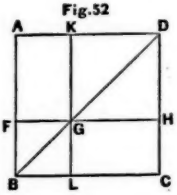
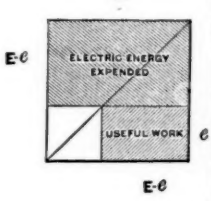


Fig.53



B D be drawn. Now measure out from the point B, along the line B A, the counter electromotive force e of the motor. The length of this quantity will increase as the velocity of the motor increases. Let e attain the value B F. Let us inquire what the actual current will be, and what the energy of it; also what the work done by the motor is. First complete the construction as follows: Through F draw F G H, parallel to B C, and through G draw K G L, parallel to A B. Then the actual electromotive force at work in the machine, and producing the current, is $E-e$, which may be represented by any one of the lines A F, K G, G H, or L C. Now the electric energy expended per second is

$$\frac{E(E-e)}{R},$$

and the work done by the motor is

$$\frac{e(E-e)}{R}$$

Since R is a constant, the relative values of the two may be written respectively:

$$E(E-e)$$

and

$$e(E-e).$$

Now

$$\text{area A F H D} = E(E-e),$$

and

$$\text{area G L C H} = e(E-e).$$

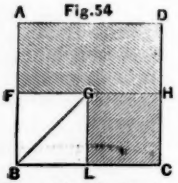
The ratio of these two areas on the diagram is the efficiency of the motor.

Let us turn to Fig. 53, in which these areas are shaded. This figure represents a case where the motor is too heavily loaded, and can turn only very slowly, so that the counter-electromotive force e is very small compared with E. Here the area, which represents the energy expended, is very large; while that which represents useful work realized in the motor is very small. The efficiency is obviously very low. Two-thirds or more of the energy is being wasted in heat.

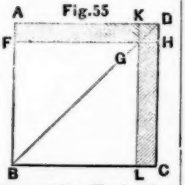
Next, let us turn our attention to the smaller area, G L C H. Obviously, the value of this area will depend upon the position of the point F; for if F is very near B, the area will be very small. If F moves upward, the area, G L C H, will have a value that increases up to a certain point, and then diminishes again; for if F be taken very near A (that is, if e be very nearly equal to E), the area, G L C H, will become again a very narrow strip. Of all the possible cases, the area of this rectangle will be a maximum when G is midway between B and D; for of all possible rectangles that can be inscribed in the triangle, B C D, that rectangle which is a true square will have the greatest value, as drawn in Fig. 54. But if G is midway between B and D, the rectangle, G L C H, will be exactly half the area of the upper rectangle, A F H D; which is, in fact, Jacobi's law of the efficiency of a motor doing its work at its greatest possible rate. Also F will be half way between B and A, which signifies that $e = \frac{1}{2}$ E.

circuit. If we want to get the greatest economy, we should group our cells so as to have an internal resistance much less than the external. We shall not get so strong a current, it is true; and we shall use up our zincs more slowly; but a far greater proportion of the energy will be expended usefully, and a far less proportion will be wasted in heating the battery cells. The maximum economy will, of course, be got by making the external resistance infinitely great as compared with the internal resistance. Then all the energy of the cement will be utilized in the external circuit, and none wasted in the battery. But it would take an infinitely long time to get through a finite amount of work in this extreme. The same kind of reasoning is strictly applicable to dynamos used as generators, the resistance of the rotating part of the circuit being the counterpart to the internal resistance of the battery cells. For good economy the resistance of the armature should be very low as compared with that of the external circuit. Mr. Edison seems to have been struck by this point, as his endeavors to reduce the resistance of his armature coils to the utmost limit of possibility show.

Again, consider these two rectangles when the point G moves indefinitely near to D (Fig. 55). We know, from Euclid's Second Book, familiar from our school-days, that the rectangle, G L C H, is equal to the rectangle, A F G K. The area (square), K G H D, which is the excess of A F H D, over A F G K, must, therefore, represent that part of the electric energy which is wasted in heating the resistance of the motor and circuit. In Fig. 53 this corner square, which stands for the heat-waste, was enormous. In Fig. 54 it was exactly half the energy. In Fig. 55 it is only about one-eighth. Clearly, we may make the heat-waste as small as we please,



Geometric illustration Jacobi's law of maximum rate of doing work.



Geometric illustration Siemens's law of efficiency.

if only we will take the point F very near to A. The efficiency will be a maximum when the heat-waste is a minimum. The ratio of the areas, G L C H and A F H D, which represents the efficiency, can only become equal to unity when the square, K G H D, becomes indefinitely small; that is, when the motor runs so fast that its counter-electromotive force, *e*, differs from E by an indefinitely small quantity only. Further, it is clear that if our diagram is to represent any given efficiency—for example, an efficiency of 90 per cent., then the point, G, must be taken so that the area G L C H = 9-10ths of the area A F H D; or G must be 9-10ths of the whole distance along B D. This involves that *e* shall be equal to 9-10ths of E, or that the motor shall run so fast as to reduce the current to 1-10th of what it would be if the motor were standing still. Thus we verify, geometrically, Siemen's law of efficiency.*

In all the preceding discussion, I have supposed that the motor is to be worked with a supply of current furnished at a fixed electromotive force. It is not only convenient, but I believe wise, to make such a condition the basis of the argument, because this is, probably, the condition under which electric power will, in the not very distant future, be distributed over large areas. It would be absurd, in the present stage of electro-technical science, to deal with such a question as the construction and use of motors, without taking into account the practical conditions under which they will be used. It is true that the condition of having a constant fixed electromotive force is not the only condition of supply; for, as we have seen in preceding lectures, a generator or system of generators may be worked so as to yield a constant current. And it would be quite possible to formulate a set of rules for the efficiency and maximum duty of motors under this condition. But this method of distributing electric power is far less likely to be of importance in the near future, than the system of distribution with constant electromotive force; though for the case of transmission of power to an isolated station, the case becomes of importance. One simple problem connected with this case is worthy of mention. Suppose that one is desirous of working a motor, so as to do work at the rate of a specified number of horse-power, and that the wire available to bring the current cannot safely stand more than a certain current, without being in danger of becoming heated unduly. It might be desirable to know what electromotive force such a motor ought to be capable of giving back, and what electromotive force must be applied at the transmitting end of the wire. Let N stand for the number of horse-power to be transmitted, and *c* for the maximum strength of current that the wire will stand (expressed in ampères.) Then, by the known rule for the work of of a current, since—

$$\frac{ec}{746} = N,$$

* We have all along supposed the motor to be a "perfect" one, that is to say, one which gave out as available mechanical work all the electric work it took from the supply of energy of the current. No motor does this, because of friction, &c. But it is easy to adapt the construction to the case. Suppose when working the motor as a generator, so as to give an electromotive force, *e*, we found that it was capable of converting 80 per cent. of mechanical energy into electrical energy; then we may put down the efficiency of the motor, *per se*, under the given conditions at 80 per cent. All we have, then, to do, is to take an area 8-10ths of the size of the area G L C H (or cut off 1-5th of G L C H), and this will really represent the available mechanical work of the motor.

$$e=\frac{746N}{c},$$

gives the condition as to what electromotive force the machine must be capable of giving, when running at the speed it is eventually to run at as a motor. Moreover, the primary electromotive force, E, must be such that

$$\frac{E-e}{\Sigma R}=c$$

where ΣR is the sum of all the resistances in the circuit. Whence,

$$E=e+c\Sigma R$$

which is the required condition.

Another problem in the application of motors to transmission of power, which vitally affects their construction, is the determination of the relation of the heat-waste to the electromotive force at which the current is supplied to the motor.

If, as before, ΣR stands for the sum of all the resistances in the circuit, then, by Joule's law, the heat-waste is (in mechanical measure)

$$HJ=C^2\Sigma R.$$

And, since $c=\frac{E-e}{\Sigma R}$ we may write the heat-waste as

$$HJ=\frac{(E-e)^2}{\Sigma R}.$$

Now suppose that without changing the resistances of the circuit we can increase E, and also increase e, while keeping $E-e$ the same as before, it is clear that the heat loss will be precisely the same as before. But how about the work done? Let the two new values be respectively E' and e'. Then the electric energy expended is—

$$W'=\frac{E'(E'-e')}{\Sigma R},$$

and the useful work done is—

$$w'=\frac{e'(E'-e')}{\Sigma R}.$$

That is to say, with the no greater loss in heating, more energy is transmitted, and more work done. Also the efficiency is greater, for

$$\frac{w'}{W'}=\frac{e'}{E'},$$

and this ratio is more nearly equal to unity than—because both E and e have

$\frac{e}{E}$

received an increment arithmetically equal. Clearly, then, it is an economy to work at high electromotive force. The importance of this matter, first pointed out by Siemens, and later by Marcel Deprez, cannot be overrated. But how shall we obtain this higher electromotive force? One very simple expedient is that of driving both the generator and motor at higher speeds. Another way is to wind the armatures of both machines with many coils of wire having many turns. This expedient has, however, the effect of putting great resistances into the circuit. This circumstance may, nevertheless, be no great drawback, if there is already a great resistance in the circuit—as, for example, the resistance of many miles of wire through which the power is to be transmitted, in this case, doubling the electromotive force will not double the resistance. Even in the case where the line resistance is insignificant, an economy is effected by raising the electromotive force. For, as may be deduced from the equations, when $E-e$ is kept constant, the effect of doubling the electromotive force is to double the efficiency, when the resistance of the line is very small as compared with that of the machines, and to quadruple it when the resistance of the line is very great as compared with that of the machines. It is, in fact, worth while to put up with the extra resistance, which we cannot avoid, if we try to secure high electromotive force by the use of coils of fine wire of many turns. It is true that the useful effect falls off, *cæteris paribus*, as the resistance increases: but this is much more than counter-balanced by the fact that the useful effect increases in proportion to the square of the electromotive force.

In the recent attempt of M. Marcel Deprez to realize these conditions, in the transmission of power from Miesbach to Munich, through a double line of telegraph wire, over a distance of thirty-four miles, very high electromotive forces were actually employed. The machines were two ordinary Gramme dynamos, similar to one another, but their usual low-resistance coils had been re-

placed by coils of very fine wire. The resistance of each machine was consequently 470 ohms, whilst that of the line was 950 ohms.* The velocity of the generator was 2,100 revolutions per minute; that of the motor, 1,400. The difference of potential at the terminals of the generator was 2,400 volts; at that of the motor, 1,600 volts. According to Professor von Beetz, the President of the Munich Exhibition, where the trial was made, the mechanical efficiency was found to be 32 per cent. M. Deprez has given the rule that the efficiency $\frac{w}{W}$ is obtained, in the case where two identical machines are employed, by comparing the two velocities at the two stations. Or

$$\frac{w}{W} = \frac{n}{N}$$

Where N is the speed of the generator, n that of the motor. There is, however, the objection to this formula, that the electromotive forces are not proportional to the speeds, unless the magnetic fields of the two machines are also equally intense, and the current running through each machine the same. This is not the case if there is leakage along the line. Moreover, when there are resistances in the line, the ratio of the two electromotive forces of the machines is not the same as the ratio of the two differences of potentials, as measured between the terminals of the machines.

I now turn back from these somewhat abstract questions to consider, by the light we have derived already, some points in the design and construction of motors. We shall find that many of the rules suggested in my former lecture are applicable also to the case of motors, and supply answers to many of the questions that naturally arise.

In the first place, shall we build motors large, or shall we build numbers of small ones? In my first lecture I proved that, in the dynamo used as a generator, the capacity for doing work increased as the fifth power of the linear dimensions; that by doubling a dynamo in length, breadth, and thickness, we had a machine weighing eight times as much, costing less than eight times as much, but capable of doing thirty-two times the work, and that with a great gain in economy in working. The same thing is true of motors. Suppose instead of building eight small motors, we build one large one, of doubled dimensions. It will not cost so much as the eight, will get through four times as much work as the eight put together, and will be more economical in working. I assume here, of course, that the large machine can be placed under equally advantageous conditions of supply.

This is by no means the only one of the points in the theory of the dynamo which can be applied to practice in the cases in which the dynamo is used as a motor. I will give another example.

In the prospect of an immediate field of usefulness opening out for motors, so soon as we have such a thing as regular town supplies of electric currents laid on, it is most important that motors should be designed, not simply to work with the constant electromotive force supplied at the electric mains, but designed also to work at uniform speeds. It is highly important, for example, in driving a lathe, and, indeed, many kinds of machinery, that the speed should be regular, and the motor should not "run away" as soon as the stress of the cutting tool is removed. Now, in my second lecture, I spent some time explaining the methods by which M. Marcel Deprez and Professor Perry had solved a converse problem to this, namely, that of getting a dynamo to feed a circuit with currents, at a constant electromotive force, when driven with a uniform speed. The solution to that problem we saw consisted in using certain combinations for the field magnets, which gave an initial magnetic field, independently of the actual current furnished by the dynamo itself. Now, it is not hard to see that this problem may be applied conversely, and that motors may be built with a combination of arrangements for their field magnets, such that, when supplied with currents at a certain constant electromotive force, their speed shall be constant, whatever the work or no work which they may be doing. The one difficulty in the problem—and this is a mere matter for experiment and calcula-

* These figures, and those which follow, are given on the authority of the President of the Munich Exhibition, Professor von Beetz. It is but fair to add that M. Deprez is not satisfied of their accuracy.

tion—is to find the critical number of volts of electromotive force at which this will hold good, It is, in fact, the converse to the operation of finding the critical velocity at which one of Deprez's or of Perry's combination dynamos must be driven, in order that it may give a constant electromotive force. M. Marcel Deprez has, himself, constructed motors upon this plan. Some three years ago, I saw one of Deprez's motors, which ran at a perfectly uniform speed, quite irrespective of the work it was doing. Whether it was lifting a load of five kilogrammes from the ground, or was letting this load run down to the ground, or ran without any load at all, the speed was the same. At the Paris Exposition Electrique of 1881, a large number of Deprez's motors were shown, running at uniform speed and driving various machines, lathes, sewing-machines, &c.

Amongst others who have aimed at producing a motor to work at uniform speed, are Professors Ayrton and Perry. Professor Ayrton informs me that one of these motors, weighing only 350 pounds, will give an effective power equal to 8-horse power, and that "without any mechanical governor; without anything, in fact, in the nature of a moving governor, it always goes at the same speed, whatever work it has to do." Certain little legal matters connected with the protection of this invention have debarred Professor Ayrton from informing me more fully of the new machine, which, if it fulfils the promise of its inventors, will be a great step forward in the mechanical application of electricity. But I can hazard a guess that the arrangement for obtaining uniform speed is dependent on the prior condition that the motor be supplied, either with a constant electromotive force, or with a constant current of a certain "critical" value; and that the solution of the problem is virtually one of the cases of combination, the counterpart of which, as applied to generators, I discussed last week.

It is, of course, possible to use as a motor any direct current dynamo, whether the field-magnets be series-wound, shunt-wound, separately excited, or permanently magnetized. There is this curious point of difference in different cases. Suppose the dynamo to be arranged so as to work as a generator, and then to be supplied with currents from an exterior source, to make it work as a motor. If the dynamo is series-wound, it will run the reverse way (or against its brushes), no matter which way the currents run through it. If the dynamo be shunt-wound, it will run with its brushes, whichever direction the current runs through it. The direction of rotation taken by the separately-excited and the magneto-machine will also be with the brushes, if the current is in the right direction, through the armature. These points have to be taken into account in any attempt to combine the different systems.

If we attempt to apply to motors rules and suggestions, such as these applied to generators in the first of these lectures, we shall find that, whilst some of them apply directly, others are singularly in contrast. For example, we found it advisable, for the sake of steadying the currents in generators, to use large and long field-magnets with plenty of iron, and with heavy pole pieces. In the case of motors, there is no such necessity laid upon us; for we want here to produce a uniform steady rotation. Now, even if the impulses be intermittent, the mechanical inertia of the moving parts will steady the motion. Electric curents have no such inertia (except in so far as the self-induction in a circuit exerts an influence like that of inertia), and hence the precautions for generators. In the case of generators, we found that, to produce steady currents, we had to multiply coils on the armature in many separate paths, grouped round a ring or a drum, involving a complicated winding, and a collecting apparatus, consisting of many segments. In motors no such necessity exists, provided only we arrange the coils that there shall be no dead points. I do not say that, for large motors, it may not be advisable to multiply the paths and segments for other reasons—as, for example, to obviate sparking at the collectors—but, for securing steady running, the inertia of the moving parts spares us—at any rate, in small machines—the complication of parts which was expedient in the generator. Some of the most successful of the little motors that have recently appeared—those, for example, of Deprez,

Trouvé, and Griscom—have for their armatures the simple old shuttle-wound Siemens armature of 1856, and in these motors, of course, there is the disadvantage of dead-points to take into account. Deprez, in his first motors, placed this armature longitudinally between the poles of a horse-shoe magnet, with the axis parallel to the limbs. He has also constructed motors with two such armatures on one spindle, one of the coils being 90° in advance of the other, so that while one was at the dead point the other should be in full action. The same suggestion has been carried out in Akestor's motor. Trouvé has tried to get over the dead-points, by utilizing the method of oblique approach mentioned earlier in my lecture. The Griscom motor, which has little copper rollers as commutator brushes, has, for field-magnets, a compact tubular electro-magnet wound in series with the armature. It has the disadvantage of dead-points. There is, in all these motors, the disadvantage that at every half-revolution the magnetism of the armature core is reversed; and as in all these forms this core is of solid iron, there must be waste by heating in the cores. In fact, to the rotating armatures of motors, as to those of generators, apply all the rules about slitting to get rid of induced eddy-currents, avoiding idle coils and useless resistances, &c. The rules about proper pole-pieces, adjustable brushes, multiplication of contacts, also are mostly applicable to motors as well as to generators.

In order to meet the case of a handy and reliable motor, I have designed a machine, the smallest size of which is on the table before you. The field-magnets, which also constitute the bed-plate of the motor, are of malleable cast iron, of a form that can be cast in one, or at most, two pieces. The form of them is that of a Joule's magnet, with large pole-pieces, and wound with coils, arranged partly in series, partly as a shunt, in certain proportions, so as to give me a constant velocity when worked with an external electromotive force of a certain number of volts.

As an armature, I employ a form which, I think, unites simplicity with efficiency for the end desired. I do not want dead points, and yet I do not want to have a complicated armature. I have, therefore, modified the old Siemens armature by embedding, as it were, one of the shuttle-shaped coils within another at right angles to one another and having duplicated the coils, I must, of course, duplicate the segments of the commutator, which, therefore, becomes either a four-part collector, or else a double collar, according to circumstances. There are no solid iron parts in the armature, but the cores are made of thin pieces of sheet iron, stamped out and strung together. Of the efficiency of this little motor you were witnesses a little while ago, when I employed it in the experiment with the galvanometer.

There are here on the table two other forms of motor, one of which is the design of Mr. A. Reckenzaun, C.E., and which is interesting because its armature, though a drum-armature in form, in reality consists of independent coils, connected, like those of the Brush dynamo, to separate commutators. There are, in fact, four commutators grouped as two twos, and the two pairs of brushes in contact with them.

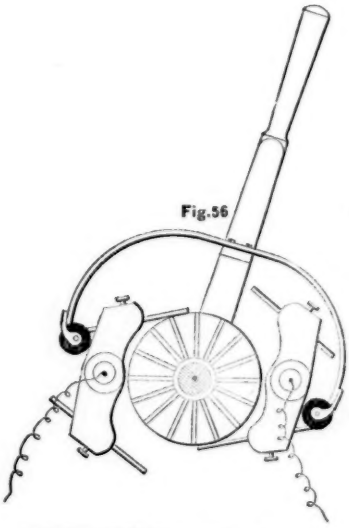
The other motor is one of the pattern invented by De Meritens, who employs a ring-armature very like that of Gramme, but places it between very compact and light field-magnets, which form a framework to the machine. There is one point about this machine of great interest, which is, however, a later addition. It is provided with a reversing gear, which, when I move the handle, reverses the direction in which it runs. This gear, which is the invention of Mr. Reckenzaun (to whom I am indebted for the loan of this motor) is shown in Fig. 56. In it there are two pairs of brushes; the two upper are fixed to a common brush-holder, which turns on a pivot, and can be tilted by pressing a lever handle to right or to left. The two lower brushes are also fixed to a holder. Against each brush-holder presses a little ebonite roller, at the end of a bent steel spring, fixed at its middle to the handle. The result of this arrangement is that, by moving the lever, the brushes can be made to give a lead in either direction, and so starting the motor rotating in either direction. Such a reversing gear is obviously a most essential adjunct for industrial applications of motors, and, if the difficulties of sparking at the brushes, caused by the sudden removals of them from the

collector, be obviated, must prove much better than any mechanical device to reverse the motion, by transferring it from the axle of the motor through a train of gearing to some other axle. One great advantage of electric motors is, that they can be so easily fixed directly on the spindle of the machine they are to drive; an advantage not lightly to be thrown away.

There is, indeed, an immense field for useful industrial application of the electric motors, so soon as we once have at our disposal regular town-supplies of electric currents. Already we have got the little motors of Griscom, Howe, and others, adapted to work sewing-machines, and instruments requiring very small power. Larger motors for driving lathes and heavier machinery, though not yet much known to the public, are in the market. Measrs. Siemens have brought out an electric lift or elevator,* in which a small dynamo, itself running very quickly, drives an endless screw, and communicates a slow but powerful motion to a drum on which the hauling chain is wound. If town-supplies of electricity were accomplished facts, such lifts would be multiplied. Indeed, there very many purposes for which hydraulic power is used, for which electric power might be advantageously substituted. The electric railway, dimly foreshadowed by George Little in 1844, when he patented an "electromotive," has become an established fact. From the electric railway first established in Berlin in 1880, by Siemens and Halske, we already have had the further developments of the electric tramway in Paris, and the experimental electric railway at the Crystal Palace. Edison has constructed an electric railway at Menlo Park, and built electric locomotives to ply upon it. Nor must I omit to mention how, in the bleaching fields of France, an electric tramway does most satisfactory duty; nor how, finally, in the sister isle, at the Giant's Causeway, Messrs. Siemens Brothers have established a line of electric railway, which is now actually in operation.

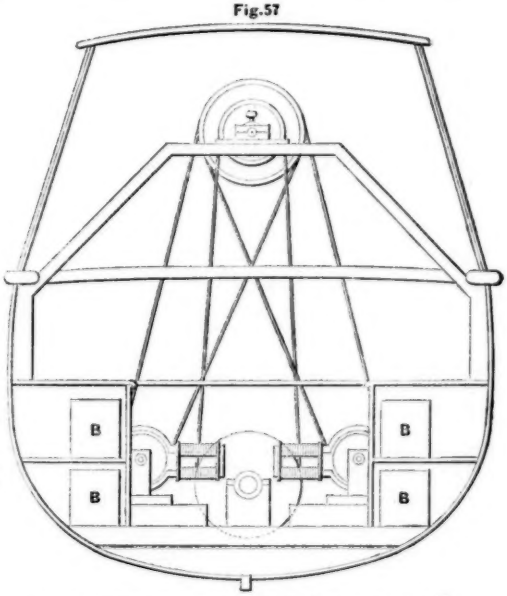
Latest of all the applications of the dynamo as a motor is that of electric

Fig.56



RECKENZAUN'S REVERSING GEAR.

Fig.57



MACHINERY OF THE LAUNCH "ELECTRICITY."

* Dr. Hopkinson has also invented an electric lift, in which the armature of the motor, running at a high speed, works the chain of the lift by a train of toothed wheels, which reduce the speed. This elevator was shown in operation at the Paris Exposition of 1881.

navigation. Jacobi, as we know, fitted up, in 1838, an electric paddle-boat, which he propelled on the Neva, first with Daniell's and afterwards with Grove's cells, and was very able when he used 128 large cells, to carry fourteen persons against the stream, at a speed of about 1¼ miles per hour.

M. Trouvé, of Paris, was the next to apply electric power, in the shape of Planté's accumulators, to a pleasure-boat, with which he navigated the Seine. But M. Trouvé has more recently abandoned storage cells, and has returned to the use of a bichromate battery, wherewith to drive the little boats, which he fits up with his little motors, mounted upon the top of the rudder, and turning a light screw-propeller by a driving belt, which passes down behind the rudder into the water.

Still more recent is the electric launch which made its trial trip on the Thames,* last September. This is an iron boat twenty-five feet long, fitted with a screw, and able to carry twelve persons, while running against the tide, at a pace of eight miles an hour. No existing motor of requisite power having yet been made of a form convenient to place directly upon the axis of the propeller, the expedient has, in this case, been resorted to of driving the screw by belts from an overhead counter-shaft, which receives its motion from two small Siemen's dynamos placed below. The arrangements of the machinery of the boat are depicted in Fig. 57. Each of these dynamos has been provided by Mr. Reckenzaun, who designed the fittings of the launch, with reversing gear like that shown in Fig. 56. The current, which actuates these dynamos, is derived from forty-five accumulators of the Sellon-Volckmar type, which are charged while the launch lies at its moorings. Though this is a great step in electric navigation, I venture to believe that it is yet but a beginning of greater things. As with steamboats so with electric-boats, we shall find that we begin with small experimental attempts, and increase in size as experience dictates. It is but a few days since I was reading, in an old volume of the *Philosophical Magazine* for the year 1801, of Symington's early little steamboat—the forerunner both of Fulton's and of Bell's—and the writer of the article hardly dared to think of this as more than an ingenious experiment, and doubted whether large vessels could ever be practically driven by steam. When I read this curious narrative, I bethought me of the trial trip of the little launch *Electricity*, and began to think that I was not so very hopelessly wrong, when I wrote in September last: "Who shall say to what proportions this latest application may not attain in the next decade?"* Could we look forward, and see with our eyes to-night the electric boats of fifty years hence, we should not, perhaps, be more surprised than those who beheld the first rude steamboat would be, if they could now rise from their graves to behold the great ocean going steamships of to-day. Indeed, when I look around, and see so many opportunities for the future application of electric motive power, so many instances in which the dynamo would supply a welcome and economical† means

* In 1856, Mr. G. E. Dering constructed, at Messrs. Searle's, an electric boat worked by a motor, in which the rotation was effected by magnets arranged within coils, like galvanometer needles, and acted on successively by currents from a battery. It succeeded, I am informed, in making headway, though at small speed, against the tide at its swiftest.

* It is well known that a no less authority than Dr. Lardner publicly pronounced it impossible for any steamboat every to carry coals enough to enable her to cross the Atlantic. It is also recorded that a very eminent public man offered to swallow the boilers of the first steamboat that should succeed in crossing the Atlantic. But he would be a bold man who would now-a-days promise to swallow the accumulators or the dynamos of the first electric boat that shall successfully make the same trip across the water.

† As to the economy of electric motive power, I may perhaps be allowed to quote a passage from a recent lecture of my own in Bristol, in which a concrete case is presented: "When it is possible thus to obtain power by merely turning on a tap, there will be an enormous impetus to all kinds of small machinery. Motors to work sewing-machines already exist, and would be used literally by millions if the source of electricity were but at hand, in the form of a friendly wire always ready to bring the needed power. . . . Again, the establishment of a town supply of electricity will tend unquestionably to develop the industries of small workshops and home workshops, where a little power is required. I have already explained how, with steam power, small engines of one horse-power or less are uneconomical. But with electricity it is not so. The smallest electro-motors may be made just as economical as are the largest. There is, moreover, another point in their favor. They may be made far smaller, lighter, more compact, and far cheaper than steam-engines of corresponding power. . . . It is certainly possible to construct an electric motor of two horse-power for ten pounds, and such a motor would be ample for the purposes of many small workshops. Why do we not, it may be asked, hear and see more of these little machines, if they are so excellent? Why do not people adopt them? The answer is simply they cannot be used until we have a central supply of electricity. I do not hesitate to affirm that such motors will be found to present an enormous economy in comparison with steam-engines of equal power. Take the case of a small workshop which wants just two horse-power to turn a few

of doing that which is now done by less advantageous means, I begin to wonder why we do not establish central supplies of electric currents for this purpose, which is surely as important as electric lighting. Here we have the possibility of introducing machinery that is portable and light, giving up power without the inconveniences and risks hitherto attending the employment of power. Surely there must be many workshops in every town where the owner would be glad of having power without the trouble and danger of a furnace, boiler, and steam-engine, or even of a gas-engine, in his place; where a boiler cannot be put for want of space, and where a steam-pipe cannot be carried because of the inevitable condensation. So thoroughly am I convinced of the great future that lies before dynamo-electric machinery, from this point of view, that I am disposed to think electric lighting a very small item by comparison.* When once we get a proper system arranged for the general distribution of electric currents in a regular town supply, we shall begin to realize the hundreds of ways, as yet undreamed of in which electricity may be utilized to meet our wants, and to subserve the ends of civilization.

lathes or drilling machines. A small steam-engine and boiler will, together, cost about £70, and the consumption of coal will be about 13 lbs. weight per hour. Instead of this, put in a motor costing, let us say, £20, which allows for handsome profits on manufacture. Here is a saving in capital outlay at once. Moreover, the motor may be put right into the workshop, for it is small—perhaps two feet long, eighteen inches broad, and one foot high—and is not liable to leak or explode. The next point to consider is the cost of running it. Let us suppose that it has only an efficiency of 70 per cent., or that it only turns 7-10ths of the energy of the current into useful mechanical work. (Many motors will do far more than this.) Let us also suppose that the currents are furnished by a dynamo whose efficiency is 90 per cent. (which figure is also capable of being attained in practice), worked by a large central steam-engine, consuming only 1½ lbs. of coal per horse-power. Allow 4 per cent. for loss, by resistance, in the conducting cables; this reduces the total efficiency of the generator to 86 per cent., and of that 86 per cent. only 7-10ths—*i. e.*, 60.2 per cent—is eventually converted back into useful work by the motor. We have, then, 40 per cent. wasted in order to get 60 per cent. of useful work. How does this affect the coal consumption for two horse-power? At the central station 3 lbs. of coal per hour would suffice were there no loss in conversion into electric energy, and back again into mechanical power. As it is, 5 lbs. of coal per horse-power will be needed, 3 lbs. of which represent the 60 per cent. of useful power transmitted, and 2 lbs. to represent the 40 per cent. wasted. In practice, I believe, a far less margin of waste might be looked for. Yet even with this large waste there is still an immense economy effected, for we get our two horse-power by an expenditure of 5 lbs. of coal at the central station, instead of 13 lbs. of coal in the furnace of a small steam-engine. It is clear, then, that electricity offers an enormous future to small motor machinery."

* What is the proportion of coal used for the manufacture of gas in Great Britain, compared with the amount consumed in procuring motive power? The answer to this query amply justifies my view.

ON THE ADJUSTMENT OF CONDITION-OBSERVATIONS IN THE METHOD OF LEAST SQUARES, WITH ITS APPLICATION TO GEODETIC WORK, AND WITH SPECIAL REFERENCE TO AMERICAN METHODS OF THE ADJUSTMENT OF A SYSTEM OF TRIANGULATION.

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I.

1. In the treatises on the method of least squares published so far in the English language the subject of condition-observations has received but slight notice. Though really a very important application of the method, it has not been systematically discussed by English or American authors mainly for the reason that their treatises were written for a special purpose, and the point aimed at stopped short of that here mentioned. Chauvenet, Airy, and Watson treated the subject chiefly so far as it related to astronomical work, while Merriman wrote professedly for the beginner. Chauvenet's treatise, the oldest and most complete, was first published twenty years ago. The systematic development of the subject of condition-observations was not made till quite recently, in fact not till Hansen published his "*Geodätische Untersuchungen*," and more especially since the publication of the *Danske Gradmaaling* by Andrae. These two works contain the only great advances made since Bessel's time. The fruit of

Andrae's and Hansen's work is seen in the very elegant methods of reduction now in use in the Prussian, Italian and other European surveys. The publications of the Great Trigonometrical Survey of India deserve special mention. Among other good points in these most admirable volumes is a record of failures as well as successes.

2. The only two surveys in America in which extended geodetic work of the highest precision has been carried on with modern instruments, are the Coast Survey and the Lake Survey. The former has been engaged for many years in mapping the Atlantic and Pacific coasts while similar work has been done by the latter on the southern shores of the great lakes. Incidentally arcs of the meridian and arcs of the parallel have been measured on both surveys and others are in progress. The recent change of name of the Coast Survey to Coast and Geodetic Survey, foreshadows a complete survey of the whole country. As an accurate triangulation lies at the basis of every survey, it cannot be out of place to lay together in a connected form the modern methods in use in reducing a triangulation and to illustrate the same from examples of actual work.

The form used on the Coast Survey is scattered through different annual reports, but will be found mainly in the reports of the years 1854, 1864, 1865 and 1868; while the form used on the Lake Survey will be found in the annual report of the Chief of Engineers for 1872, and also in the final report of the Lake Survey lately published.

3. As the adjustment of a triangulation is only a special case of condition-observations, I shall for greater clearness show first how these are treated, and then pass to the special problem, taking up first the rigorous method of adjustment, and next the Lake Survey and Coast Survey forms. I shall also give the Hansen-Schreiber method which is in many respects the best yet proposed, and conclude by deriving a set of rules for adjusting a system of triangulation which can be applied by any one who does not care to learn the method of least squares.

The reader is assumed to be familiar with Chauvenet's treatise on the method of least squares. I shall, as far as possible, use Chauvenet's notation and nomenclature. Some minor variations will be noticed such as "mean-square error" for "mean error," "observation-equation"* for "equation of condition" and the like.

4. If a series of observed quantities are not independent, that is if they must satisfy exactly certain relations imposed in some way outside of the observations they are said to be *conditioned* by these relations.

If a series of quantities whether directly observed or functions of observed quantities are independent of one another the method of least squares shows that we may pick out the most probable system of values from all possible systems by making the weighted sum of the squares of the residual errors of the observed quantities a minimum.

There is really no new principle introduced in the treatment of condition-observations. It is merely an extension of the general principle just stated and all that is done is to apply various devices for getting over difficulties and for shortening work demanded by the straightforward process.

Thus, if

$$F = f(x\ y\ z \ldots) \qquad (1).$$

is a function of n variables $x, y, \ldots$ and is to be made a minimum subject to the n_e conditions

$$\begin{aligned} \varphi_1 (x, y, z \ldots) &= 0 \\ \varphi_2 (x, y, z \ldots) &= 0 \qquad (2). \\ &\ldots\ldots\ldots\ldots \end{aligned}$$

it is plain that n_e of the variables can be expressed in terms of the remaining $n - n_e$ and that by substituting these values in the original equation we have to make a minimum the resulting function.

$$F = \psi (x, y, z \ldots .)$$

containing $n - n_e$ independent variables.

The $n - n_e$ equations resulting from differentiating this equation with reference to the $n - n_e$ independent variables, taken in connection with the n_e equations of condition determine the n quantities $x, y, z \ldots$. This is the direct solution of the problem and may be used with advantage when the condition-equations are simple.

The solution of the equations contain-

* On this phrase see some good remarks by Merriman, *Least Squares*, London, 1877.

ing the $n-n_e$ independent unknowns may be carried through as in Chauvenet, p. 544, &c., though the forms there given are not always the best for computation.

5. In general, however, the elimination here indicated is carried out by the method of undetermined multipliers. Call k', k'', . . . the multipliers of the condition-equations in order. Then the function

$$f(x, y, z, \ldots) = a \text{ minimum},$$

is determined by making a minimum the equivalent function,*

$$f(x, y, z \ldots) + k'\varphi_1(x, y, z \ldots) + k''\varphi_2(x, y, z \ldots) + \ldots \quad (3).$$

with reference to the $n-n_e$ independent variables.

The differential coefficients for the $n-n_e$ independent variables will then be equal to zero and to determine the n_e multipliers k', k'' . . . we may assume the differential coefficients for the n_e variables not independent also equal to zero. Hence we have in all the n $(=\overline{n-n_e} + n_e)$ equations

$$\frac{df}{dx}=0$$

$$\frac{df}{dy}=0$$

. . . . . .

together with the n_e condition-equations from which to determine the $n+n_e$ unknown quantities.

The multipliers k', k'' . . . are called by Gauss the *correlates* of the condition-equations.

6. For simplicity I shall divide the practical consideration of the problem into two parts.

1. When the condition-equations are solved simultaneously.

2. When the condition-equations are solved in groups.

1.—*The Simultaneous Solution of the Condition-Equations.*

7. It is a general principle that functions of observed quantities must first be reduced to the linear form before being treated by the method of least squares.

Let then V_1, V_2, . . . be the most probable values of n directly observed quantities M_1, M_2 . . . whose weights are p_1, p_2 . . . respectively. Let the n condition-equations to be exactly satisfied by the most probable values of the observed quantities when reduced to the linear form be

$$a'V_1 + a''V_2 + \ldots - L' = 0$$
$$b'V_1 + b''V_2 + \ldots - L'' = 0 \quad (4).$$
. . . . . .

If v_1, v_2 . . . denote the most probable corrections to the observed values, then

$$V_1 - M_1 = v_1$$
$$V_2 - M_2 = v_2 \quad (4\tfrac{1}{2}).$$
. . . . . .

and we have the reduced condition-equations

$$a'v_1 + a''v_2 + \ldots - l' = 0$$
$$b'v_1 + b''v_2 + \ldots - l'' = 0 \quad (5).$$
. . . . . .

where

$$l' = L' - [aM]$$
$$l'' = L'' - [bM].$$
. . . . . .

The most probable system of corrections is that which makes

$$[pv^2] = a \text{ minimum},$$

that is which makes a minimum (see Art. 5)

$$(a'k' + b'k'' + \ldots)v_1 + (a''k' + b''k'' + \ldots)v_2 + \ldots \tfrac{1}{2}(p_1v_1^2 + p_2v_2^2 + \ldots) \quad (6).$$

with reference to v_1, v_2 . . as independent variables.

Hence differentiating we have the n equations

$$a'k' + b'k'' + \ldots - p_1v_1 = 0$$
$$a''k' + b''k'' + \ldots - p_2v_2 = 0 \quad (7).$$
. . . . . .

from which with the n_e condition-equations to determine the quantities k', k'' . . . v_1, v_2 . . .

Substituting for v_1 v_2 . . . in the condition-equations their values derived from the above equations we have n_e equations containing n_e unknowns k', k''. They are the normal equations

$$\left\{\frac{aa}{p}\right\}k' + \left\{\frac{ab}{p}\right\}k'' + \ldots - l' = 0$$
$$\left\{\frac{ab}{p}\right\}k' + \left\{\frac{bb}{p}\right\}k'' + \ldots - l'' = 0 \quad (8).$$
. . . . . .

Solving three equations we obtain k', k'' . . . and thence v_1, v_2 . . . from (7) and V_1, V_2 . . . from $(4\tfrac{1}{2})$.

Hence the problem is solved.

* See Todhunter's Diff. Calc., Arts. 238, 239.

8. The form of the normal equations shows that it is convenient to use the reciprocals of the weights instead of the weights themselves. They may be written

$$\begin{aligned}[uaa]k'+[uab]k''+\ldots-l'=0\\ [uab]k'+[ubb]k''+\ldots-l''=0\end{aligned}\quad(9).$$

$$\cdots\cdots\cdots\cdots$$

when u_1 u_2 . . . denote the reciprocals of the weights p_1, p_2 . . .

Hence in computing the coefficients of the normal equations it is of great service to first of all write down from the condition-equations a tabular form as follows:

		k'	k''	k'''
v_1	u_1	a'	b'	c'
v_2	u_2	a''	b''	c''
v_3	u_3	a'''	b'''	c'''

$$\cdots\cdots\cdots\cdots$$

9. If the elimination of equations (9) is performed by the method of substitution (Chauvenet, § 42) we have by collecting the first equations of the successive groups,

$$\begin{aligned}[uaa]k'+[uab]k''+[uac]k'''+\ldots-l'=0\\ [ubb1]k''+[ubc1]k'''+\ldots-l'1=0\\ [ucc2]k'''+\ldots-l'''2=0\end{aligned}\quad(10).$$

$$\cdots\cdots\cdots\cdots$$

the notation being analogous to the ordinary Gaussian form. Divide each of these equations by the first unknown and we have

$$\begin{aligned}k'+\frac{[uab]}{[uaa]}k''+\frac{[uac]}{[uaa]}k'''+\ldots-\frac{l'}{[uaa]}=0\\ k''+\frac{[ubc1]}{[ubb1]}k'''+\ldots-\frac{l'1}{[ubb1]}=0\\ k'''+\ldots-\frac{l'''2}{[ucc2]}=0\\ +\ldots\ldots\ldots\end{aligned}\quad(11).$$

These equations being precisely similar in form to Chauvenet's equations (70), using the quantities, R', R'' . . . S'' . . . corresponding to his A', A'', . . . B'' . . . we have, as in §§ 48, 49,

$$\begin{aligned}k'=\frac{l'}{[uaa]}+\frac{l'1}{[ubb1]}R'+\frac{l'''2}{[ucc2]}S''+\ldots\\ k''=\quad+\frac{l'1}{[ubb1]}+\frac{l'''2}{[ucc2]}S''+\ldots\\ \cdots\cdots\cdots\cdots\\ l'1=l'R'+l'\ldots\ldots\ldots\\ l'''2=l'R''+l''S''+l'''\ldots\\ \cdots\cdots\cdots\cdots\end{aligned}\quad(12).$$

and

$$\begin{aligned}[uaa]=\frac{1}{[uaa]}+\frac{R'^2}{[ubb1]}+\frac{R''^2}{[ucc2]}+\ldots\\ [ua\beta]=\quad\frac{R'}{[ubb1]}+\frac{R'S''}{[ucc2]}+\ldots\\ \cdots\cdots\cdots\cdots\\ [u\beta\beta]=\quad\frac{1}{[ubb1]}+\frac{S''^2}{[ucc2]}+\\ [u\beta\gamma]=\quad\frac{S''}{[ucc2]}+\ldots.\\ \cdots\cdots\cdots\cdots\end{aligned}$$

where $[u.aa]$, $[ua\beta]$. . satisfy the relations

$$\begin{aligned}[u.aa][.uaa]+[u.ab][u.a\beta]+\ldots=1\\ [u.ab][u.aa]+[u.bb][u.a\beta]+\ldots=0\\ [u.aa][u.a\beta]+[u.ab][u.\beta\beta]+\ldots=0\\ [u.ab][u.a\beta]+[u.bb][u.\beta\beta]+\ldots=1\end{aligned}\quad(13).$$

$$\cdots\cdots\cdots\cdots$$

10.—*To find the precision of the adjusted values.*

The first step is to find the mean-square error μ of an observation of weight unity.

In Chauvenet, Art. 37, it is shown that in a system of observation-equations the mean-square error μ of an observation of the unit of weight is found from

$$\mu=\sqrt{\frac{[pvv]}{n-n_i}},$$

where

$[pvv]$ is the sum of the weighted squares of the residual errors v, n is the number of observation-equations and n_i the number of independent unknowns.

Hence in a system of condition-observations, n being the number of observed quantities and n_e the number of conditions, the number of independent unknowns is $n-n_e$, and

$$\begin{aligned}\mu&=\sqrt{\frac{[pvv]}{n-(n-n_e)}},\\ &=\sqrt{\frac{[pvv]}{n_e}}\ldots\ldots\ldots\end{aligned}\quad(14).$$

The m. s. e. of an observation of weight p is

$$\mu\sqrt{u}\ldots\ldots\ldots\quad(15).$$

where u is the reciprocal of p.

Hence to find the m. s. e. of any adjusted value we must make two distinct computations one of μ and one of u.

11.—*Checks of* $[pvv]$.

When the number of residuals is large, in order to guard against mistakes $[pvv]$

should be computed in at least two different ways. The following check method will be found convenient.

(1). Equation (7) may be written

$$\sqrt{p_1}.v_1=\sqrt{u_1}a'k'+\sqrt{u_1}b'k''+\ \ldots\ .$$
$$\sqrt{p_2}.v_2=\sqrt{u_2}a''k'+\sqrt{u_2}b''k''+\ \ldots\ .$$
$$\ldots\ldots\ldots\ldots$$

Square and add, and

$$[pvv]=[u.aa]k'k'+2[uab]k'k''+2[uac]k'k'''+\ \ldots$$
$$+[ubb]k''k''+2[ubc]k''k'''+\ \ldots$$
$$+[ucc]k'''k'''+\ \ldots$$
$$+\ \ldots$$
$$=[kl]\text{ from (11) }\ldots\ldots\ (16).$$

(2) $[pvv]=kl$

$$=l'_1k'+l''k''+\ \ldots\ldots\ldots$$
$$=l'\left(\frac{l'}{[uaa]}+\frac{l''1}{[ubb1]}\mathrm{R}'+\frac{l'''2}{[u.cc2]}\mathrm{R}''+\ldots\right)$$
$$+l''\left(\frac{l''1}{[ubb1]}+\frac{l'''2}{[ucc2]}\mathrm{S}''+\ \ldots\right)$$
$$+\ \ldots\ldots\ldots\ldots$$
$$=\frac{l'^2}{[uaa]}+\frac{[l''1]^2}{[ubb1]}+\frac{[l''2]^2}{[ucc2]}+\ \ldots\ldots\ (17).$$

by addition attending to equations (12).

This is very readily computed from the solution of the correlate normal equations (11).

12.—*To find the weight and mean-square error of any function of the adjusted values* $\mathrm{V}_1, \mathrm{V}_2, \ldots$.

Let the function be

$$\mathrm{F}=f(\mathrm{V}_1, \mathrm{V}_2, \ldots\ldots)$$

and let it be conditioned by the equations

$$\varphi_1(\mathrm{V}_1, \mathrm{V}_2, \ldots\ldots)=0$$
$$\varphi_2(\mathrm{V}_1, \mathrm{V}_2, \ldots\ldots)=0$$
$$\ldots\ldots\ldots\ldots$$

Expressing F in terms of the values which are independent of one another and reducing to the linear form, we have

$$d\mathrm{F}=\frac{d\mathrm{F}}{d\mathrm{M}_1}.d\mathrm{M}_1+\frac{d\mathrm{F}}{d\mathrm{M}_2}d\mathrm{M}_2+\ \ldots$$

Hence

$$\mu^2_{\mathrm{F}}=\left(\frac{d\mathrm{F}}{d\mathrm{M}}\right)^2\mu_1^2+\left(\frac{d\mathrm{F}}{d\mathrm{M}_2}\right)^2\mu_2^2+\ \ldots\ (19).$$

when $\mu_1, \mu_2, \ldots$ are the m. s. e. of $\mathrm{M}_1, \mathrm{M}_2, \ldots$ respectively.

As it usually requires a very long elimination to get F expressed directly in terms of $\mathrm{M}_1, \mathrm{M}_2, \ldots$ it is better to compute $\frac{d\mathrm{F}}{d\mathrm{M}_1}, \frac{d\mathrm{F}}{d\mathrm{M}_2}, \ldots$ from the forms

$$\frac{d\mathrm{F}}{d\mathrm{M}_1}=\frac{d\mathrm{F}}{d\mathrm{V}_1}.\frac{d\mathrm{V}_1}{d\mathrm{M}_1}+\frac{d\mathrm{F}}{d\mathrm{V}_2}.\frac{d\mathrm{V}_2}{d\mathrm{M}_1}+\ \ldots\ .$$
$$\frac{d\mathrm{F}}{d\mathrm{M}_2}=\frac{d\mathrm{F}}{d\mathrm{V}_1}.\frac{d\mathrm{V}_1}{d\mathrm{M}_2}+\frac{d\mathrm{F}}{d\mathrm{V}_2}.\frac{d\mathrm{V}_2}{d\mathrm{M}_2}+\ \ldots\ .$$
$$\ldots\ldots\ldots\ldots$$

13. It is, however, usually much more convenient in practice to proceed as follows:

Let the function reduced to the linear form be

$$\mathrm{F}=f'v_1+f''v_2+\ \ldots\ldots\ldots\ (20).$$

This is conditioned by the equations

$$a'v_1+a''v_2+\ \ldots\ -l'=0$$
$$b'v_1+b''v_2+\ \ldots\ -l''=0$$
$$\ldots\ldots\ldots\ldots$$

with $[pv^2]=$a minimum.

Referring to the principle of Art. (5) we find by using the correlates $k', k'' \ldots$ that we can express the function F in terms of all the quantities $v_1, v_2 \ldots$ which must now be taken as independent. Hence

$$d\mathrm{F}=(f'-a'k'-k'k''-\ \ldots\ )v_1+(f''-$$
$$a''k'-b''k''-\ \ldots\ )v_2+\ \ldots$$
$$=\mathrm{F}'v_1+\mathrm{F}''v_2+\ \ldots\text{ suppose,}\qquad (21).$$

and therefore

$$u_{\mathrm{F}}=[u.\mathrm{FF}]\ \ldots\ldots\ldots\ (22).$$

the required result.

It remains now to find $k', k'' \ldots$ Calling k the correlate of equation (21) we have from this condition-equation and the minimum equation

$$k\mathrm{F}'-p_1v_1=0$$
$$k\mathrm{F}''-p_2v_2=0$$
$$\ldots\ldots\ldots\ldots$$

Hence

$$[u\mathrm{FF}]=\left\{\frac{\mathrm{FF}}{p}\right\}=[pv^2]=\text{a minimum,}$$

and therefore $k', k'' \ldots$ are found from the following equations which are of the form of observation equations:

$$a'k'+b'k''+\ \ldots\ -f'=\mathrm{F}'\text{ weight }u_1$$
$$a''k'+b''k''+\ \ldots\ -f''=\mathrm{F}''\quad\text{“}\quad u_2$$

The resulting normal equations are

$$[uaa]k'+[uab]k''+\ \ldots\ -[uaf]=0$$
$$[uab]k'+[ubb]k''+\ \ldots\ -[ubf]=0\qquad (23).$$
$$\ldots\ldots\ldots\ldots\ \ldots$$

These equations being precisely of the form of Chauvenet equations (65) we have, proceeding as in Art. 44,

$$u_{\mathrm{F}}=[uff]-\left\{\frac{[uaf]^2}{[uaa]}+\frac{[ubf1]^2}{[ubb1]}+\right\}.\ (24).$$

which formula will be found of very convenient application in problems of this kind. The formula itself is due to Gauss.

14. The correlate equations (23) being the same as the regular correlate equations in all butthe constant term, we may find u_F from the following scheme in which (uaf), (ubf) . . are added as an extra column in the solution of the correlate equations. Three correlates are taken,

k'	k''	k'''			
(uaa)	(uab)	(uac)	$-l'$	$-(uaf)$	
	(ubb)	(ubc)	$-l''$	$-(ubf)$	
		(ucc)	$-l'''$	$-(ucf)$	(uff)
	$(ubb1)$	$(ubc1)$	$-l'1$	$-(ubf1)$	
		$(ucc1)$	$-l''1$	$-(ucf1)$	$(uf1)$
		$(ucc2)$	$-l''2$	$-(ucf2)$	$(uff2)$
					$(uff3)=u_F$

15.—*To find the average value of the ratio of the weight of an observed quantity to that of its adjusted value in a system of condition observations.*

Here

$$F=V_1.$$

Hence, since

$$f_1'=1, f''=f'''=\ldots=0$$
$$[uaf]=u_1a_1$$
$$[ubf]=u_1b_1.$$
$$\ldots\ldots\ldots$$

and using the forms in Chauvenet, Art. 49, we have

$$[ubf1]=(R'a'+b')u_1$$
$$[ucf2]=(R''a+s''b'+c')u_1$$
$$\ldots\ldots\ldots\ldots$$

Substituting these values in

$$u_F=u_1-\frac{u_1^2a'^2}{[uaa]}+\frac{u_1^2(R'a'+b')^2}{[ubb1]}+\frac{w_1^2(R''a'+s'b'+c')^2}{[ucc.2]}+\ldots$$

or,

$$\frac{p_1}{P_1}=\frac{u_p}{u_1}=1-\left\{\frac{u_1a'^2}{uaa}+\frac{u_1(R'a'+b')^2}{ubb1}+\frac{u_1(R''a'+s''b'+c')^2}{[ucc2]}+\ldots\right\}$$

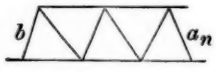
where P_1 is the weight of the adjusted value of V_1.

Now, summing the corresponding expressions for all of the quantities from V_1 to V_n, we have, after expanding and arranging,

$$\left[\frac{p}{P}\right]=n-[uaa]\left\{\frac{1}{[uaa]}+\frac{R'^2}{[ubb1]}+\frac{R''^2}{[ucc2]}+\ldots\right\}$$
$$-[uab]\left\{\frac{R'}{[ubb1]}+\frac{R''s''}{[ucc2]}\ldots\right\}\ldots$$
$$-[uab]\left\{\frac{R'}{[ubb1]}+\frac{R''s''}{[ucc2]}+\ldots\right\}-[ubb]\left\{\frac{1}{[ubb1]}+\frac{s''^2}{[ucc2]}+\ldots\right\}\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$=n-[uaa][uaa]-[uab][ua\beta]-\ldots$$
$$-[uab][ua\beta]-[ubb][u\beta\beta]-\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$=n-n_c$$

=the number of observations — the number of conditions (25). This result was, far as I know, first published in Vol. 2 of the G. T. Survey of India. It has been used in the precision determination of the Lake Survey triangulation.

16. A special case of the general theorem of Art. 12 of frequent occurrence is the finding of the m. s. e. of a triangle side in a triangulation chain, all of the angles of each triangle having been measured.



Let b be the measured value of the base and let a_1 a_2 . . . A_n be the sides of continuation in order as computed from b, a_n being the side whose m. s. e. is required.

If A_1, B_1 . . . are the measured values of the angles used in computing the terminal side from the base, B_1, B_2 . . . being opposite to the bases and A_1 A_2 . . opposite to the sides of continuation, then

$$\frac{a_1}{b}=\frac{\sin.A_1}{\sin.B_1},\ \frac{a_2}{a_1}=\frac{\sin.A_2}{\sin.B_2},\ \ldots,\ \frac{a_n}{a_{n-1}}=\frac{\sin.A_n}{\sin.B_n}$$

Hence, by multiplication,

$$a_n=b.\frac{\sin.A_1.\sin.A_2.\ldots.}{\sin.B_1.\sin.B_2.\ldots.} \qquad (26).$$

We may proceed in two ways:

(*a*) Differentiating directly,

$$da_n=\frac{a_n}{b}.db+a_n.\cot.A_1\sin1''.dA_1-a_n\cot.B_1\sin.1''.dB_1+\ldots.$$

$$= \frac{a_n}{b}(b) + a_n \sin.1''$$

$$\Big(\text{cot. A (A)} - \text{cot. B(B)}\Big) \quad \ldots \quad (27).$$

where (A), (B) . . . denote the corections dA, dB, . . . This expression is of the standard form (20).

(*b*) Taking logs of both members of (26) and then differentiating,

$$d \log. a_n = d \log. b + \text{mod. cot. A}_1 \sin. 1'' \, d\text{A}_1 - \text{mod. cot. B}_1 \sin. 1'' \, d\text{B} + \ldots$$

$$= (\log. b) + [\delta_{\text{A}}(\text{A}) - \delta_{\text{B}}(\text{B})].$$

where δ_{A}, δ_{B}, . . are the log. differences corresponding to 1″ for log. sin. A, log. sin. B, . . in a table of log sines.

This expression is also of the standard form (20). The coefficients of the corrections in these equations taken in connection with those of the condition-equations when substituted in the general formula (19) give the relation required. Having found the mean square error of log. a_n we easily find that of a_n. For

$$d \log. a_n = \frac{\text{mod.}}{a_n} da_n.$$

Hence,

$$\mu^2 \log. a_n = \left(\frac{\text{mod.}}{a_n}\right)^2 \mu^2 a_n.$$

The logarithmic form is the better of the two in practice.

Example 1.—A single triangle.

Here $$a = b \frac{\sin. \text{A}}{\sin. \text{B}},$$

$$\therefore \quad da = \frac{a}{b}(b) + a \sin.1''[\text{cot.A(A)} - \text{cot. B(B)}].$$

First consider the base exact, that is, omit the first term.

We have the condition equation

$$(\text{A}) + (\text{B}) + (\text{C}) = l$$

$$\therefore a' = a'' = a''' = 1.$$

Also,
$$f' = a \text{ cot. A. sin. } 1''$$
$$f'' = -a \text{ cot. B. sin. } 1''$$
$$f''' = 0.$$

Hence if p_1, p_2, p_3 denote the weights of the measured angles,

$$[uaa] = u$$
$$[uaf] = u_1 a \text{ cot. A. sin.} 1'' - u_2 a \text{ cot. B. sin.} 1''$$
$$[uff] = u_1 a^2 \text{cot.}^2\text{A. sin.}'' - u_2 a^2 \text{ cot.}^2 \text{B. sin.}'',$$

and

$$v_a = [uff] - \frac{[uaf]^2}{[uaa]}$$

$$= a^2 \Big(_1\text{cot.}^2\text{A} + u_2 \text{cot.}^2\text{B} - \frac{(u_1 \text{cot.A} - u_2 \text{cot. B})^2}{[u]} \Big) \sin.^2 1'' \quad (29).$$

If μ is the m. s. e. of a measured angle coresponding to the unit of weight, then

$$\mu_a = \mu \sqrt{v_a}$$

$$= \mu a \sin.1''$$

$$\sqrt{u_1 \text{cot.}^2\text{A} + u_2 \text{cot.}^2\text{B} - \frac{(u_1 \text{cotA} - u_2 \text{cot. B})^2}{[u]}}$$

Also if μ_b denotes the m. s. e. of the base b the expression for $\mu^2 a$ must be increased by

$$\frac{a^2}{b^2} \cdot \mu^2 b.$$

If the weights p_1, p_2, p_3 are equal, then

$$\mu^2 a = \frac{a^2}{b^2} \cdot \mu^2 b + \frac{2}{3} a^2 \mu^2 \sin.^2 1'' (\text{cot.}^2\text{A} + \text{cot. }^2\text{B} + \text{cot. A .cot.B}) \quad (30).$$

and if the triangle is equilateral

$$\mu^2 a = \mu^2 b + \frac{2}{3} a^2 \mu^2 \sin.^2 1'' \quad (31).$$

Example 2.—Next consider a chain of triangles, the base being free from error, and the corresponding angles of the several triangles of the same weight.

We have

$$a_1 = b \frac{\sin. \text{A}_1}{\sin. \text{B}_1} = b\lambda_1 \text{ suppose}$$

$$a_2 = a_1 \cdot \frac{\sin. \text{A}_2}{\sin. \text{B}_2} = a_1 \lambda_2 = b \lambda_1 \lambda_2$$

$$\ldots\ldots\ldots\ldots$$

$$a_n = b . \lambda_1 \lambda_2 \ldots \lambda_n$$

By differentiation

$$da_n = a_n \left(\frac{d\lambda_1}{\lambda_1} + \frac{d\lambda_2}{\lambda_2} + \ldots\ldots \right)$$

$$\therefore \quad \mu^2 a_n = a^2 n \left(\frac{\mu^2 \lambda_1}{\lambda_1^2} + \frac{\mu^2 \lambda_2}{\lambda_2^2} + \ldots. \right)$$

But from

$$\lambda \mu^2 \lambda_1 = \mu^2 \Big(u_1 \text{cot.}^2\text{A}_1 + u_2 \text{cot.}^2\text{B}_1 - \frac{(u_1 \text{cot.} \text{A}_1 - u_2 \text{cot. B}_1)^2}{[u]} \Big) \sin. 1'',$$

and since the corresponding angles of the triangles are of the same weight, we have

$$\frac{\mu^2 \gamma_1}{\lambda_1^2} = \frac{\mu^2 \lambda_2}{\lambda_2^2} = \ldots.$$

Hence,

$$\mu^2_{a_n} = n.a^2_n \frac{\mu^2_\lambda}{\lambda^2} \qquad (32).$$

that is, in a chain of similar and similarly measured triangles $\frac{\mu_{a_n}}{a_n}$ increases as the square root of the number of triangles in the chain.

If the chain consists of equilateral triangles and the weights of the angles are each equal to unity, then

$$\mu_a = \sqrt{\tfrac{2}{3}}.\ \text{sin. } 1''.\mu b\sqrt{n}$$
$$= 0.000004\mu b\sqrt{n} \text{ nearly.} \qquad (33).$$

Hence in a chain of equilateral triangles the weights of the sides decrease as we proceed from the base through the successive triangles, as

$$1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \ldots$$

If in a chain of equilateral triangles the m. s. e. of the base is μ_b, then

$$\mu^2_a = \mu^2 b + \tfrac{2}{3}\mu^2 \text{ sin.}^2 1''.b^2 n \qquad (34).$$

Now in the measurement of a primary base a precision may be attained such that the m. s. e. of the length does not exceed $\frac{1}{1,000,000}$ part of the length. The m. s. e. of angle of the triangulation may be taken as $0''.25$, and therefore the m. s. e. of a side computed through n equilateral triangles is

$$0.000004 \times 0.25\sqrt{n}$$
$$= 0.000001\sqrt{n}$$

part of the side.

Hence it only takes a very few triangles from the base in order that the error arising from the angles exceeds that from the base, and in computing a long chain the base error may be neglected, that is, the measurement of the base may be considered perfect in comparison with the measurement of the angles.

17. A very important principle as to the best form of triangle ABC to be employed in order to reach the greatest precision in the length of a side a computed from another side b as base, may be derived from the equation.*

$$\mu_a = \mu a \text{ sin.}'' \sqrt{\tfrac{2}{3}(\text{cot.}^2 \text{A} + \text{cot.}^2 \text{B} + \text{cot. A cot. B})}.$$

when μ is the m. s. e. of an observed angle.

* Zachariæ, Die geodätische Hauptpunkte, Berlin, 1878.

If the side c is determined with the same precision as a then,

$$\frac{\mu_c}{c} = \frac{\mu_a}{a},$$

and therefore C=A

and B=180−2A.

Hence,

$$\mu_a = \mu a \text{sin. } 1'' \sqrt{\frac{2}{3}} \cdot \frac{3 + 4 \text{ tan.}^4 \text{A}}{4 \text{ tan.}^2 \text{A}}.$$

The precision is greatest when this expression is a minimum, that is, when

$$\text{A} = 52°\ 46',$$

and then $\mu_a = \frac{3.7}{10^6}.\mu_a$.

With an equilateral triangle on the other hand, $\mu_a = \frac{4.0}{10^6}.\mu_a$.

Since this value of μ_a is greater than the preceding, we might conclude that the above isosceles triangle was the normal form.

But in a chain of triangles similar to this triangle, each would be smaller than the preceding. Thus, since

$$\frac{a}{b} = \frac{\text{sin.} 52°46'}{\text{sin.} 74°28'} = 0.83,$$

each triangle is 0.7 of the preceding one in the chain, and therefore with a long chain but little progress would be made.

On the other hand, in a chain of equilateral triangles a greater area is covered than in a chain of the same number of scalene triangles. Hence, since with equilateral triangles there is least loss of precision with the greatest covering of ground, we conclude that the equilateral is the normal form.

Solution of the Condition Equations in Groups.

18. The preceding solution is simple, straightforward, and rigorous. It has the serious drawback of being in general very laborious. When the number of condition equations is large, as in a triangulation adjustment, the amount of labor required to make a direct solution would be enormous. Accordingly, methods have been devised of solving the equations in groups. I shall notice two of these: the first, which is due to Bessel, is rigorous; and the second, due to Gauss,

leads to results as accurate as we please through successive approximations.

Bessel's Method of Solution.

19. In many kinds of work it happens that the conditions to which observations are subject may be divided into two classes for purposes of solution. The division is arbitrary, as may be seen, for example, in Bessel's and Schleiermacher's* solutions of the same problem in triangulation. Bessel's method, which is the more general, consists in solving the first set of condition-equations by themselves and then utilizing the work in determining the further connections due to the remaining conditions. In this way the work is kept in manageable shape.

Thus let the reduced observation-equations n in number and containing n_u unknowns be

$$\begin{aligned} a_1x+b_1y+\ldots.-l_1&=v_1 \text{ weight } p_1\\ a_2x+b_2y+\ldots.-l_2&=v_2 \quad\text{“}\quad p_2 \end{aligned} \quad (35).$$

. . . . . .

and the condition-equations n_c in number, involving the same unknowns be

$$\begin{aligned} a'x+a''y+\ldots.-l'&=0\\ b'x+b''y+\ldots.-l''&=0 \end{aligned} \quad (36).$$

. . . . . .

The most probable values of $x, y, \ldots$ are those which satisfy the condition

$$[pv^2]=\text{a minimum}$$

it is required to find them.

20. The total correction to the value of an unknown may be divided into two parts:

1. That depending on the observation-equations.

2. That depending on the condition-equations.

Denote the first by the correction in parenthesis and the second by the numbers 1, 2, . . . also in parenthesis, thus

$$\begin{aligned} \text{Total correction } x&=(x)+(1)\\ \text{“}\quad\text{“}\quad y&=(y)+(2) \end{aligned} \quad (37).$$

Now overlooking the condition-equations and taking the observation-equations only $(x), (y)\ldots$ would be found by solving these equations in the usual way. We have, therefore, by reducing all to the same weight for convenience in writing, the normal equations,

$$\begin{aligned} [aa](x)+[ab](y)+\ldots.-[al]&=0\\ [ab](x)+[bb](y)+\ldots.-[bl]&=0 \end{aligned} \quad (38).$$

. . . . . .

The solution of these equations gives (see Chauvenet, equation 50).

$$\begin{aligned} (x)&=[aa][al]+[a\beta][bl]+..\\ (y)&=[a\beta][al]+[\beta\beta][bl]+. \end{aligned} \quad (39).$$

. . . . . .

Again, substituting for $v_1, v_2 \ldots$ in the minimum equation, we find

$$\begin{aligned} [aax]x+2[ab]xy+\ldots.-2[al]x\\ +\ [bb]yy+\ldots.-2[bl]y \end{aligned} \quad (40).$$

. . . . . .

$$+[ll]=\text{a minimum.}$$

This is conditioned by (36). Hence the solution is the same as in Art. 7.

Calling I, II . . . the correlates of equations (36), we have

$$\begin{aligned} [aa]x+[ab]y+.-[al]-a'\text{I}-b''\text{II}-.&=0\\ [ab]x+[bb]y+.-[bl]-a''\text{I}-b''\text{II}-.&=0 \end{aligned}$$

. . . . . .

These equations taken with (37) and (38) give the relations

$$\begin{aligned} [aa](1)+[ab](2)+.&=a'.\text{I}+b'\text{II}+..\\ &=[1] \text{ suppose}\\ [bb](2)+..&=a''\text{I}+b''\text{II}+..\\ &=[2] \text{ suppose} \end{aligned} \quad (41).$$

. . . . . .

Since these equations are of the same form as (38) their solution gives

$$\begin{aligned} (1)&=[aa][1]+[a\beta][2]+\ldots\\ (2)&=[a\beta][1]+[\beta\beta][2]+. \end{aligned} \quad (42).$$

. . . . . .

or substituting for [1], [2], . . their values from (41)

$$\begin{aligned} (1)&=\text{A}'\text{I}+\text{B}'\text{II}+\text{C}'\text{III}+..\\ (2)&=\text{A}''\text{I}+\text{B}''\text{II}+\text{C}''\text{III}+. \end{aligned} \quad (43).$$

. . . . . .

where

$$\begin{aligned} \text{A}'&=[aa]a'+[a\beta]a''+..\\ \text{B}'&=[aa]b'+[a\beta]b''+.. \end{aligned} \quad (44).$$

. . . . . .

We have ∴ expressed the condition-corrections in terms of the correlates. Substituting for $x, y \ldots$ their values from (37) in the condition-equations and we find

$$\begin{aligned} a'(1)+a''(2)+\ldots.-l'_0&=0\\ b'(1)+b''(2)+\ldots.-l''_0&=0 \end{aligned} \quad (45).$$

. . . . . .

where

$$\begin{aligned} -l'_0&=a'(x)+a''(y)+..-l'\\ -l''_0&=b'(x)+b''(y)+..-l'' \end{aligned} \quad (46).$$

. . . . . .

*See Fischer's Geodasie, Part III.

The values of l_0', l_0'' . . are known since (x), (y) . . are known.

Substitute the values of (1), (2) . . from (43) in (45) and we have the correlate normal equations,

$$[\overline{aA}]\text{I}+[\overline{aB}]\text{II}+\ldots-l_0'=0$$
$$[\overline{aB}]\text{I}+[\overline{bB}]\text{II}+\ldots-l_0''=0 \quad (47).$$
$$\ldots\ldots\ldots\ldots$$

where

$$[\overline{aA}]=[\overline{aa}]a'a'+[\overline{a\beta}]a'b'+\ldots$$
$$+[\overline{a\beta}]a'b'+[\overline{\beta\beta}]b'b'+\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$\&c.=\&c.$$

The solution of these equations gives the correlates I, II . . .

21. In carrying the preceding solution into practice, the following order of procedure will be found convenient:

(1). The formation and solution of the observation-equations (35). The partially adjusted resulting values (x), (y) . . . are now to be used.

(2). The formation of the condition-equations.

$$a'(1)+a''(2)+\ldots-l_0'=0$$
$$b'(1)+b''(2)+\ldots-l_0''=0$$
$$\ldots\ldots\ldots\ldots$$

(3). The formation of the weight-equations. They are at once written down from the general solution of the observation-equation equations, and are

$$(1)=[aa][1]+[a\beta][2]+\ldots$$
$$(2)=[a\beta][1]+[\beta\beta][2]+\ldots$$
$$\ldots\ldots\ldots\ldots$$

(4). The formation of the correlate-equations,

$$[1]=a'.\text{I}+b'\text{II}+\ldots$$
$$[2]=a''\text{I}+b''\text{II}+\ldots$$
$$\ldots\ldots\ldots\ldots$$

(5). The expression of the corrections in terms of the correlates by substituting from (4) in (3),

$$(1)=\text{A}'.\text{I}+\text{B}'\text{II}+\ldots$$
$$(2)=\text{A}''\text{I}+\text{B}''\text{II}+\ldots$$

(6). The formation of the normal equations by substituting from (5) in (2),

$$[\overline{aA}]\text{I}+[\overline{aB}]\text{II}+\ldots-l_0'=0$$
$$[\overline{aB}]\text{I}+[\overline{bB}]\text{II}+\ldots-l_0''=0$$
$$\ldots\ldots\ldots\ldots$$

(7). The determination of the corrections by substituting the values of the correlates in (5).

22.—*To find the m. s. e. of an observation of weight unity.*

We have

$$\mu^2=\frac{[vv]}{n-(n_u-n_c)} \quad (48).$$

where n is the number of observed quantities and n_u-n_c the number of independent unknowns. In practice it is better to write the denominator in the form

$$(n-n_u)+n_c,$$

$n-n_u$ being the number of superfluous observation-equations and n_c the number of condition-equations.

Let v_1^0, v_2^0 . . . denote the residual errors arising from taking the observation-equation only, then

$$v_1^0=a'(x)+b'(y)+\ldots-l'$$
$$v_2^0=a''(x)+b''(y)+\ldots-l'' \quad (49).$$
$$\ldots\ldots\ldots\ldots$$

and from the normal equations (38)

$$[av^0]=[bv^0]=\ldots=0$$

that is (see Chauvenet p. 512),

$[v^0v^0]$ is formed in the usual way from the observation-equations.

Again,

$$v_1=a_1[(x)+(1)]+b_1[(y)+(2)]+\ldots$$
$$=v_1^0+a_1(1)+b_1(2)+\ldots$$
$$v_2=v_2^0+a_2(1)+b_2(2)+\ldots \quad (50).$$
$$\ldots\ldots\ldots\ldots$$

Hence,

$$[vv]=(v^0v^0)+[(a(1)+b(2)+\ldots)^2]+$$
$$2(av^0)(1)+2(bv^0)(2)+\ldots$$
$$=[v^0v^0]+[\text{WW}] \text{ suppose.} \quad (51).$$

where

$$[\text{WW}]=[(a(1)+b(2)+\ldots)^2]$$
$$=(1)[(aa)(1)+(ab)(2)+\ldots]$$
$$+(2)[ab(1)+(bb)(2)+\ldots]$$
$$+\ldots\ldots\ldots\ldots\ldots\ldots$$
$$=(1)[1]+(2)[2]+\ldots\ldots \quad (52).$$

from (41).

Substitute for (1) [1], (2) [2] . . . their values from equations 41, 43 and expand: then

$$[\text{WW}]=[[\overline{aA}]1+[\overline{aB}]11+\ldots]\text{I}$$
$$+[\overline{aB}]1+[\overline{bB}]11+\ldots]\text{II}$$
$$+\ldots\ldots\ldots\ldots\ldots$$

which may be transformed as in Art. 9 into the form

$$[\text{WW}]=\frac{l_0'^2}{[\overline{aA}]}+\frac{l_0''^2}{(\overline{bB1})}+\frac{l_0'''^2}{[\overline{cC2}]}+\ldots \quad (53).$$

or from equation (47) into the form

$$[\text{WW}]=l_0'\text{I}+l_0''\text{II}+\ldots \quad (54).$$

23.—*To find the weight of a function of the adjusted values.*

Let the function reduced to the linear form be

$$d\mathrm{F}=g_1x+g_2y+\ldots. \qquad (55).$$

Put for x, y,... their values $(x)+(1)$, $(y)+(2)$,... then

$$d\mathrm{F}=g_1(x)+g_2(y)+..-g_1(1)+g_2(2)+..$$

Substitute for (1), (2)..their values from (43) and

$$d\mathrm{F}=g_1(x)+g_2(y)+..+[g\mathrm{A}]\mathrm{I}+[g\mathrm{B}]\mathrm{II}+.$$

where I, II,..satisfy the equations

$$\begin{aligned}[a\mathrm{A}]\mathrm{I}+[a\mathrm{B}]\mathrm{II}+..-l_0'&=0\\ [a\mathrm{B}]\mathrm{I}+[b\mathrm{B}]\mathrm{II}+..-l_0''&=0\end{aligned} \qquad (47).$$

.....

Hence using the undetermined multipliers k_1, k_2.. in order to eliminate the correlates I, II,... we have as in (3)

$$\begin{aligned}d\mathrm{F}=&g_1(x)+g_2(y)+..l_0'k_1+l_0''k_2+..\\ &+[(g\mathrm{A})-(a\mathrm{A})k_1-(a\mathrm{B})k_2-..]\mathrm{I}+\\ &[(g\mathrm{B})-(a\mathrm{B})k_1-(b\mathrm{B})k_2..]\mathrm{II}+.. \qquad (56).\end{aligned}$$

We may determine k_1, k_2..so as to satisfy

$$\begin{aligned}[a\mathrm{A}]k_1+[a\mathrm{B}]k_2+..-[g\mathrm{A}]&=0\\ [a\mathrm{B}]k_1+[b\mathrm{B}]k_2+..-[g\mathrm{B}]&=0\end{aligned} \qquad (57).$$

.....

and then

$$d\mathrm{F}+g_1(x)+g_2(y)+\ldots l_0'k_1+l_0''k_2+. \qquad (58).$$

Substitute for l_0', l_0''..their values from (46) and

$$\begin{aligned}d\mathrm{F}=&g_1(x)+g_2(y)+..+k_1[l'-a'(x)-\\ &b'(y)\ .]+k_2[l''-a''(x)-b''(y)..]+..\\ =&(lk)+(x)(g_1-a'k_1-a''k_2..)+(y)\\ &(g_2-b'k_1-b''k_2..)+....\\ =&(lk)+\mathrm{G}_1(x)+\mathrm{G}_2(y)+..\text{suppose.} \qquad (59).\end{aligned}$$

where

$$\begin{aligned}\mathrm{G}_1&=g_1-a'k-a''k_2-..\\ \mathrm{G}_2&=g_2-b'k_1-b''k_2-..\end{aligned} \qquad (60).$$

.....

Since (x), (y)..satisfy the equations

$$\begin{aligned}[aa](x)+[ab](y)+..&=[al]\\ [ab](x)+[bb](y)+..&=[bl]\\ &\ldots\ldots+\ldots.\end{aligned} \qquad (38).$$

the problem is reduced to Art. 12.

Hence if U is the reciprocal of the required weight,

$$\mathrm{U}=[\mathrm{GQ}] \qquad (61).$$

when

$$\begin{aligned}\mathrm{Q}_1&=[aa]\mathrm{G}_1+[a\beta]\mathrm{G}_2+..\\ \mathrm{Q}_2&=[a\beta]\mathrm{G}_1+[\beta\beta]\mathrm{G}_2+..\end{aligned} \qquad (62).$$

.....

Putting for G_1, G_2.. their values from (61), we find

$$\begin{aligned}\mathrm{Q}_1&=q_1-\mathrm{A}_1'k_1-\mathrm{B}_1'k_2-..\\ \mathrm{Q}_2&=q_2-\mathrm{A}''k_1-\mathrm{B}''k_2-..\end{aligned} \qquad (63).$$

.....

when

$$\begin{aligned}q_1&=[aa]g_1+[a\beta]g_2+..\\ q_2&=[a\beta]g_1+[\beta\beta]g_2+.\end{aligned} \qquad (64).$$

.....

Hence substituting for G_1, G_2.. Q_1, Q_2.. their values,

$$\begin{aligned}[\mathrm{GQ}]=&[gq]-[g\mathrm{A}]k_1-[g\mathrm{B}]k_2-..\\ &-[aq]k_1-[bq]k_2-..\\ &+[[a\mathrm{A}]k_1+[a\mathrm{B}]k_2+.]k_1\\ &+[[a\mathrm{B}]k_1+[b\mathrm{B}]k_2+.]k_2\\ &+\ldots\ldots\ldots\ldots\end{aligned} \qquad (65).$$

But from (44) and (64)

$$[aq]=[g\mathrm{A}],\ [bq]=[g\mathrm{B}],..$$

Hence attending to equation (57) the above expression reduces to

$$[\mathrm{GQ}]=[gq]-[g\mathrm{A}]k_1-[g\mathrm{B}]k_2.. \qquad (66).$$

24. We may, however, put equation (66) in a form more convenient for computation.

For substitute for $[g\mathrm{A}]$, $[g\mathrm{B}]$,.. their values from (57) and

$$\begin{aligned}[\mathrm{GQ}]=&[gq]-[[a\mathrm{A}]k_1+[a\mathrm{B}]k_1+..]k_1\\ &-[[a\mathrm{B}]k_1+[b\mathrm{B}]k_2+..]k_2\\ &-\ldots\ldots\ldots\ldots\end{aligned}$$

which may be reduced to the form (see Art. 13),

$$[\mathrm{GQ}]=[gq]-\frac{[g\mathrm{A}]^2}{[a\mathrm{A}]}-\frac{[g\mathrm{B}1]^2}{[b\mathrm{B}1]}-\frac{[g\mathrm{C}2]^2}{[c\mathrm{C}2]}-.. \qquad (67).$$

25. The form of the last expression for $[\mathrm{GQ}]$ suggests that it may be found from the following scheme, adding $[\mathrm{A}g]$, $[\mathrm{B}g]$, ... as an extra column in the solution of the normal equations. It is analogous to Art. 15.

I.	II.	III.		
$[a\mathrm{A}]$	$[a\mathrm{B}]$	$[a\mathrm{C}]$	$[\mathrm{A}g]$	
	$[b\mathrm{B}]$	$[b\mathrm{C}]$	$[\mathrm{B}g]$	
		$[c\mathrm{C}]$	$[\mathrm{C}g]$	$[gq]$

.....

Also

$$\begin{aligned}\mu_{\mathrm{F}}&=\mu\sqrt{\mathrm{U}}\\ &=\mu\sqrt{[\mathrm{GQ}]}\end{aligned} \qquad (71).$$

where μ is the m. s. e. of an observation of weight unity.

26.—*Check of* (gq).

From equations (64) by transposition

$$\begin{aligned}g_1&=[aa]q_1+[ab]q_2+..\\ g_2&=[ab]q_1+[bb]q_2+..\end{aligned} \qquad (72).$$

.....

Hence multiplying in order by q_1, q_2.. and adding, we have

$$[gg]=[aa]q_1q_1+2[ab]q_1q_2+..\\ +[bb]q_2q_2+...\\ .....\\ =\frac{l^2}{[aa]}+\frac{l'1^2}{[bb1]}+\frac{l''2^2}{[cc2]}+.. \quad (73).$$

when $[aa]$, $[bb1]$,... are found from the observation-equations.

2.—Gauss' Method of Solution by Successive Approximation.

27. It may be stated as follows:

A large number of condition-equations being given for solution by the method of least squares we may divide these equations into groups, and adjust the groups separately in any order we please, when the values found from the successive groups will be closer and closer approximations to the most probable values as found from the simultaneous adjustment of all of the groups.

For suppose we have the condition-equations

$$\begin{matrix}a'v_1+a''v_2+..-l_a=0\\ b'v_1+b''v_2+..-l_b=0\\ .....\\ h'v_1+h''v_2+..-l_h=0\\ k'v_1+k''v_2+..=l_k=0\\ .....\end{matrix} \quad (74).$$

with

$$[pv^2]=\text{a min.}$$

Let v_1', v_2'.. be the values of v_1, v_2... obtained from solving the first group alone, that is from

$$\begin{matrix}a'v_1'+a''v_2'+..-l_a=0\\ b'v_1'+b''v_2'+..-l_b=0\\ .....\\ pv^{2'}]=\text{a min.}\end{matrix} \quad (75).$$

If now (v_1'), (v_2').. are the corrections to these values resulting from the remaining equations, then since

$$(pv^2)=[p[(v'+(v')]^2]\\ =[p\,v'^2]+[(pv')^2]$$

The condition-equations are reduced to

$$\begin{matrix}a'(v_1')+a''(v_2')+..-l'_a\\ b'(v_1')+b''(v_2')+..-l'_b\\ .....\\ h'(v_1')+h''(v_2')+..-l'_h\\ k'(v_1')+k''(v_2')+..-l'_k\\ .....\end{matrix} \quad (76).$$

with $[p(v')^2]=\text{a min.}$

and the values of (v') found from the simultaneous solution of these equations added to the values of v' found from the solution of the first set would be equal to the value of v found directly. Similarly if v_1'', v_2''.. be the values of (v') obtained by solving the second set alone and (v_1''), (v_2'')... be the corrections to these values resulting from the remaining equations, then since

$$pv^2=[pv'^2]+[pv''^2]+[p(v'')^2]$$

the condition-equations are reduced to

$$\begin{matrix}a'(v'')+a''(v'')+..-l''_a\\ b'(v'')+b''(v'')+..-l''_a\\ .....\\ h'(v'')+h''(v'')+..-l_h\\ k'(v'')+k''(v'')+..-l_k\\ .....\end{matrix} \quad (77).$$

with $[p(v'')^2]=\text{a min.}$

The quantities $[pv'^2]$, $[pv''^2]$, ... being positive, the minimum equation is reduced with the solution of each set, and thus we gradually approach the most probable set of values. Beginning with the first set a second time and solving through again we should reduce the minimum equation still farther, and by continuing the process we shall finally reach the same result as that obtained from the rigorous solution. In practice the first approximation is in general close enough.

It is plain that the most probable values can be found after any approximation, by solving simultaneously the whole of the groups, using the values already found as approximations to the most probable values.

28. As an example we may cite the reduction of the principal triangulation of the English Ordnance Survey. Here there were 920 condition-equations, and therefore if the whole were to be reduced at once the solution of 920 equations would be necessary. The following was the method employed: "The triangulation was divided into a number of parts or figures, each affording a not unmanageable number of equations of condition. One of these being corrected or computed independently of all the rest, the corrections so obtained were substituted (so far as they entered) in the equations of condition of the next figure and the sum of the squares of the remaining corrections in that figure made a minimum.

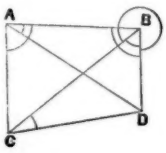
The corrections thus obtained for the second figure were substituted in the third, and so on."*

ADJUSTMENT OF A TRIANGULATION.

29. As already stated this is a special case of the method of condition-observations. I shall treat an example according to the strict principles already laid down and then pass on to the American methods, which are more or less approximate, in some of their parts at least.

The preliminary work for the formation of the condition-equations must first be explained. This will be given in some detail and will contain the results of considerable research and experience in the subject.

30. In a triangulation there must be at least one measured base as AB. Starting from this and measuring the angles CAB, ABC, we may compute the sides AC, BC by ordinary trigonometry. In



plotting the figure the point C can be located in but one way. Having only the necessary measurements there are no contradictions and therefore no room for adjustment.

Similarly by measuring the angles CBD, DCB, we may compute the sides DB, DC and the point D can be plotted in but one way.

If, however, while occupying the station A, the observer had read the angle BAD, then in plotting the figure we should find in almost all cases that the lines AD, CD, BD would not intersect in the same point. In computing the side BD or CD from AB, we should find different values according to the triangles through which we passed.

If additional angles were measured the case would be more complex. Thus if the whole angle ABD had been measured, we should have a contradiction arising from the fact that the measured value of ABD would not be equal to the sum of the measured values of ABC, CBD.

So if the blunt angle DBA had also been measured, we should have another contradiction arising from the non-satisfaction of the relation

$$ABC + CBD + DBA = 360°.$$

And not only this. For in any triangle we have considered so far, that only two angles were measured. If in the first triangle the angle BCA were also measured, we know from spherical geometry that the three angles should satisfy the relation

$$CAB + ABC + BCA = 180 + \text{sph. excess of triangle},$$

which the measured angles will not do.

So with the other triangles.

The number of contradictions to be removed will depend on the measurements made. The more unsystematic these have been the greater will be that number. We can conceive that the angles can be so changed as to satisfy all the conditions required, and that this can be done in an infinite number of ways. The method of least squares will show us how to pick out the most probable values from this infinite number of possible values.

31. In a triangulation then with one measured base in which the sides are to be computed from this base, using the most probable values of the angles, we conclude that these angles must be adjusted for two classes of conditions:

(1.) Those arising at each station from the relations of the angles to one another at that station. These are known as *local* conditions and the failure of the observed angles to satisfy these conditions arise mainly from imperfect elimination of instrumental errors and errors of observation.

(2.) Those arising from the geometrical conditions demanded by the figure,

(*a*) that the sum of the angle of each triangle shall be equal to 180° increased by the spherical excess of the triangle.

(*b*) that the length of any side as computed from the base is the same whatever route is chosen.

These are known as *general* conditions, and the failure of the observed angles to satisfy these conditions arises mainly from errors of observation and when the

* Account of the Main Triangulation p. 272.

lines are short from imperfect centering of instrument and when the lines are long from horizontal refraction, &c.

The general statement of the method of solution is this: Adjust the angles so as to satisfy simultaneously the local and general conditions. The method of least squares shows that of all possible systems of corrections to the observed quantities which satisfy these conditions, the most probable is that which makes the sum of the squares of these corrections a minimum.

32. The form of the reduction depends on the method employed in making the observations. To make this clear we shall explain the two principal methods of measuring angles, confining ourselves to non-repeating theodolites read with microscopes. These are the method of independent angles and the method of directions, an angle being the difference of two directions.

(1.) *The method of independent angles.* This method was developed by Gauss,* and may be used either with a repeating or a non-repeating theodolite.

The mode of measurement is the same as in the method of directions, the number of signals read to in any case being limited to two.

(2.) *The method of directions.* This was recommended by Gauss, but its developement is mainly due to Bessel, Hansen and Andrae.

The telescope being in position, it is directed to a signal and the micrometers read. Similar pointings and readings are made to each of the signals in the order that they are ranged round the horizon. The telescope is now reversed, that is, it is turned 180° in the azimuth and 180° in altitude leaving the same pivots in the same wyes, and the observations are made in the reverse order, beginning with the last signal and ending with the first. These readings differ from the former corresponding ones by 180° nearly. The means from the two positions of the telescope give the directions of the signals relative to one of them taken as an initial station by subtracting the mean for the initial station from the mean for each of the others. These means are free from errors arising from lack of collimation of the instrument, from inequality in the heights of the wyes, and from any uniform twisting of the instrument on its support.

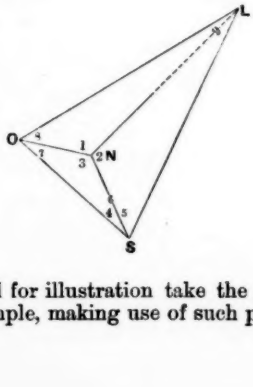
The reading of the limb on the first signal is changed by some aliquot part of the distance between consecutive microscopes, if their number is even, whenever the telescope is transitted, the same number of means being obtained on each of the equal arcs making up that distance. If the number of microscopes is odd, the circle should be shifted by an aliquot part of the half distance between consecutive microscopes after every two sets of measures with the telescope direct and reversed, the same number of measures being obtained on each of the equal arcs making up that half distance.

A single series of means is called an "arc" (*satz: mise*). It is usually the case that the conditions are such that it is not possible to measure complete *arcs* only. To save time, the stations are sighted at just as they happen to be visible, the initial station being changed as often as may be necessary. This, though economy in the field, causes a great amount of extra work in the reduction, as we shall see presently.

33. One or other of the above methods of measuring angles with slight modifications has been used by all of the leading modern surveys. The method of directions is followed by the Coast Survey and by most of the European Surveys. The Lake Survey method is a modified form of this. See Arts. 59 *seq.* for the Lake Survey method and Arts. 68 *seq.* for the Coast Survey method.

The Method of Independent Angles.

34. As the case of independent angles is the simplest to reduce, we shall begin with it.



I shall for illustration take the following example, making use of such parts of

* Astrom. Nachr., 2117.

it from time to time as may properly belong to the subject in hand.

In the triangulation of Lake Superior the following angles were measured in the quadrilateral N. Base, S. Base, Lester, Oneota.

		°	′	″		
ONL = M_1	=	124	09	40.69	weight	2
LNS = M_2	=	113	39	05.07	"	2
SNO = M_3	=	122	11	15.61	"	14
OSN = M_4	=	23	08	05.26	"	23
NSL = M_5	=	47	31	20.41	"	6
OSL = M_6	=	70	39	24.60	"	7
NOS = M_7	=	34	40	39.66	"	31
LON = M_8	=	43	46	26.40	"	1
SLO = M_9	=	30	53	30.81	"	8

The length of the line N. Base—S. Base is 6056^m.6 and the latitudes of the four stations are

N. Base	46°	46′
S. Base	46	43
Lester	46	52
Oneota	46	45

The Local or Station-Adjustment.

35. When in a system of triangulation the horizontal angles read at a station are adjusted for all of the conditions existing among the angles at the station then these angles are said to be locally adjusted.

From the considerations set forth in 31 it is readily seen that at a station only two kinds of conditions are possible,

(*a*) that an angle can be formed from two or more others, and

(*b*) that the sum of the angles round the horizon should be equal to 360°. The second of these is included in the first and the method of adjustment may be stated in general terms as follows:

An inspection of the figure representing the angles at the station will show how all of the measured angles can be expressed in terms of a certain member of them which are independent of one another. It is to be kept in mind that angles at a station may be observed independently of one another and yet not be independent of one another in the reduction.

If then M_1, M_2,... denote the single measured angles and $v_1, v_2, \ldots$ their most probable corrections then if any of them M_h, M_k can be formed from others we have by equating the measured and computed values the conditions

$$\begin{aligned} M_h + v_h &= M_1 + v_1 + M_2 + v_2 + \ldots \\ M_k + v_k &= M_1 + v_1 + M_2 + v_2 + \ldots \\ &\cdots\cdots\cdots\cdots \end{aligned}$$

or

$$\begin{aligned} v_1 + v_2 + \ldots - v_h &= l_h \text{ suppose} \\ v_1 + v_2 + \ldots - v_k &= l_k \quad " \\ &\cdots\cdots\cdots\cdots \end{aligned} \qquad (78).$$

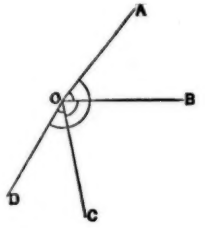
with

$$p_1 v_1^2 + p_2 v_2^2 + \ldots + p_h v_h^2 + p_k v_k^2 + \ldots = \text{a minimum},$$

where p_1, p_2,... denote the weights of the angles.

The solution may be carried out by the method of correlates as in Chauvenet, Art. 55. The following special cases are of frequent occurrence:

(1). At a station O the $m-1$ single angles AOB, BOC,... are measured and also the sum — angle AOL, to find the adjusted values of the separate angles all of them being of the same weight.



The condition-equation is

$$M_1 + v_1 + M_2 + v_2 + \ldots + M_{m-1} + v_{m-1} = M_m + v_m$$

or

$$v_1 + v_2 + \ldots + v_{m-1} - v_m = M_m - (M_1 + M_2 + \ldots + M_{m-1}), \quad = l$$

with $(v^2) = $ a minimum.

The solution gives

$$v_1 = v_2 = \ldots = -v_m = \frac{l}{m},$$

that is, the correction to each angle is $\frac{1}{m}$ of the discrepancy and the sign of the correction to the sum angle is opposite to that of the single angles.

(2). At a station O the m single angles AOB, BOC,... LOA are measured thus closing the horizon, to find the adjusted values of the angles.

The condition-equation is

$$v_1+v_2+\ldots+v_m=360-(M_1+M_2+\ldots+M_m)=l \text{ suppose,}$$

with $[pv^2]$=a minimum.

The solution gives

$$v_1=u_1\cdot\frac{l}{[u]}$$

$$v_2=u_2\cdot\frac{l}{[u]},$$

$$\ldots\ldots\ldots\ldots$$

where $u_1=\frac{1}{p_1}$, $u_2=\frac{1}{p_2}$.. and $[u]=\left[\frac{1}{p}\right]$.

If the weights are equal, then

$$v_1=v_2=..=\frac{l}{m},$$

that is, the correction to each angle is $\frac{1}{m}$ of the discrepancy.

Example.—The angles at station N. Base close the horizon: required to adjust them.

We have

M_1+v_1	$=124\ 09\ 40.69+v_1$	weight 2
M_2+v_2	$=113\ 39\ 05.07+v_2$	" 2
M_3+v_3	$=122\ 11\ 15.61+v_3$	" 14
sum	$360\ 00\ 01.37+v_1+v_2+v_3$	
Theoret. sum	$360\ 00\ 00.00$	

$\therefore$ Horizon eq. is $0 = 1.37+v_1+v_2+v_3$.

Hence,

$$v_1=-\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}+\frac{1}{14}}\times 1.37,$$

$$=-0.64$$

$$v_2=-0.64$$

$$v_3=+0.09$$

and the adjusted angles are

		° ′ ″
		124 09 40.05
		113 39 04.43
		122 11 15.52
Check sum	=	360 00 00.00

Example.—Precisely as in the preceding we may deduce at station South Base,

Measured Angles. ° ′ ″		Weights.		Adjusted Angles. ° ′ ″
23 08 05.26	..	23	..	23 08 05.13
47 31 20.41	..	6	..	47 31 19.91
74 39 24.60	..	7	..	70 39 25.04

THE GENERAL ADJUSTMENT.

36. With a single measured base the number of conditions arising from the geometrical relations existing among the different parts of a triangulation net can be readily estimated. For if the net contains s stations two are known being the end points of the base and $s-2$ are to be found.

Now since two angles determine a point, $2(s-2)$ are necessary to fix the $s-2$ points. Hence if n is the total number of measured angles, the number of superfluous ones, that is the number of conditions is

$$n-2s+4.$$

37.—*The Angle Equations.*

The sum of the angles of a triangle drawn on a plane surface is equal to 180°. The sum of the angles of a spherical triangle exceeds 180° by the spherical excess of the triangle, which latter is found from the relation*

$$\varepsilon=\frac{\text{area of triangle}}{R^2.\ \text{sin. } 1''},$$

R being the radius of the sphere.

From extended surveys carried on during the past two centuries the earth has been found to be nearly spheroidal in form and its dimensions have been determined pretty closely.

Now a spheroidal triangle of moderate size may be computed as a spherical triangle on a tangent sphere whose radius is

$$\sqrt{\rho_1\rho_2},$$

when ρ_1, ρ_2 are the radii of curvature of the meridian and of the normal section perpendicular to the meridian respectively, at the point corresponding to the mean of the latitudes φ of the triangle vertices.

Hence we may wrap one triangulation on the spheroid in question by conforming it to the spherical excess computed from the formula

$$\varepsilon \text{ (in seconds)}=\frac{ab\ \text{sin.C}}{2\rho_1\rho_2\ \text{sin.}1''} \quad . \quad (79).$$

when $a.b$ are two sides and C is the included angle of the triangle. For convenience of computation we may write

$$\varepsilon=\lambda.\ ab\ \text{sin.C}$$

when log. λ may be tabulated for the argument φ.

*Todhunter's Spherical Trigonometry, Art. 106.

To find a, b, φ, a preliminary geodetic computation must first be made of the triangulation to be adjusted, starting from a base or from a known side. The values found from using the unadjusted angles will be close enough for finding ε. The latitudes need only be computed to the nearest minute.

Any gross error may be checked by the relation:

Spherical excess in seconds $= \frac{1}{200}$ area of triangle in sq. kilometers.

Another useful check results from the principle that the sums of the excesses of triangles covering the same figure should be equal.

In our example the spherical excesses of the triangles ONS, SNL, ONL, will be found to be 0″.06, 0″.19, 0″.12, respectively.

In each single triangle then the condition required to wrap it on the spheroid, that is, that the sum of the three measured angles shall be equal to 180° together with the spherical excess gives a condition-equation. This is called an *angle-equation*, or by some a *triangle-equation*.

Example.—In the triangle N. Base, S. Base, Oneota, if v_1, v_2, v_3, denote the corrections to the three angles, we have for the most probable values:

°	′	″
122	11	15.61 + v_1
23	08	05.26 + v_2
34	40	39.66 + v_3
180	00	00.53 + v_1 + v_2 + v_3
Theoretical sum 180	00	00.06

and the angle-equation is

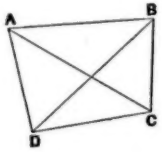
$$0 = 0.47 + v_1 + v_2 + v_3$$

There is no other relation independent of this existing among the angles, and if one side is taken as base the other sides can be found from it in but one way.

38. Complex figures must be treated by considering their component triangles. Thus, let us take the quadrilateral ABCD, in which all right angles are measured. If the spherical excesses of the triangles ABC, BCD, CDA, DAB, be computed, we have the relations:

Sum of angles of

$$\begin{aligned} ABC &= 180 + \varepsilon_1 \\ BCD &= 180 + \varepsilon_2 \\ CDA &= 180 + \varepsilon_3 \\ DAB &= 180 + \varepsilon_4 \\ ABCD &= 360 + \tfrac{1}{2}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4) \end{aligned}$$



Only three of these conditions however are independent, since from any three the others necessarily follow. The three independent conditions may be selected from the five conditions in

$$\frac{5.4.3}{1.2.3} = 10$$

ways, of which however only 8 can be used, since the two

ABD, BCD, ABCD
ADC, ABC, ABCD

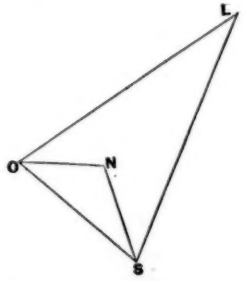
contain really two and not three different conditions.

The eight sets of polygons which can be chosen are:

ABD,	ABD,	ABD,	BAD,
ABC,	ABC,	ADC,	BCD,
ADC,	ABCD,	ABCD,	BAC,
	BCD,	CBD,	CBD, DBA
	BAC,	CAB,	CDA, DCB
	ABCD,	CDA,	ABCD, DCA

In selecting the three conditions it is in general most convenient to neglect those containing ABCD.

As another example let us take our Lake Superior quadrilateral. If all of the



angles had been measured any three of the four triangles would have furnished

independent angle-equations. But as the line NL has not been sighted over from L, we have only two angle-equations, namely, those from the triangles ONS and OLS, just the same as if the figure had been of the form.

39. It is to be expected that in a triangulation net some of the lines will be measured over in both directions, and some in only one direction. If these latter are omitted the number of angle-equations will remain the same. In general we may estimate the number of independent angle-equations in any net as follows:

If s_1 is the number of stations occupied, the polygon forming the outline of the net will give one angle-equation. Each diagonal will give an angle-equation. Hence, if there are l_1 lines in all the number of diagonals will be $l_1 - s_1$ and the *number of angle-equations*

$$l_1 - s_1 + 1 \quad . \quad . \quad . \quad . \quad . \quad (80).$$

REPORTS OF ENGINEERING SOCIETIES.

ENGINEERS' CLUB OF PHILADELPHIA. Record of regular meeting, March 17th, 1883.

President Henry G. Morris in the Chair. Mr. Chas. A. Ashburner read a paper on "A New Method of Estimating the Contents of Highly Plicated Coal Beds as Applied to the Anthracite Fields of Pennsylvania." The questions of the future production and ultimate exhaustion of these fields, are of the greatest importance. In 1860 the population of the United States was 31,443,321, and 8,513,123 tons of coal were produced, *i. e.*, actually shipped to market; in 1870 the population had increased 22 per cent. (38,558,371) and the production of anthracite was dearly doubled, being 16,182,191 tons. In 1880, with a population of over 50 millions, the product was 23,437,242+tons. In 1882 the actual production was over 30,000,000 tons. It has been variously estimated that the 470 square miles containing this coal in Pennsylvania, will be entirely exhausted in from 140 to 204 years. While Mr. Ashburner does not estimate the ultimate exhaustion, he has devised a method for estimating the contents of these fields, from data now being obtained by the careful and practical geological and mining examinations of the State Survey. The exact position and detailed structural shape of the coal beds are first mapped by 50 feet contour lines along the floor of the beds, giving, completely and satisfactorily, their geometrical construction and shape. These surfaces are then developed into planes, by the development into straight lines of the line of the bed as cut by paralleled section planes 1600 feet apart. This graphical method is attended with errors which are mathematically discussed, and which have been formulated by Mr. Arthur Winslow, Member of the Club. This method does not give the true area of the surface of a sphere, cone or triangular trough. In the case of a sphere, it gives $\frac{\pi}{4}$ of the true area; in a cone, the error increases directly as the secant of the angle which the pitch of the cone makes with its axis; and in a triangular trough, which more nearly represents the shape of the anthracite basins, the error is very much less. A practical test has been made of this method in the Panther Creek basin, between Mauch Chunk and Tamaqua, and the maximum possible error in estimating the surface area of the coal beds was found to be .905 of 1 per cent. After the areas are thus found, the contents are obtained by careful measurements made in the mines to ascertain the actual number of tons of coal which are contained in a unit (1 acre) of bed area. In this way it has been estimated that the above basin originally contained 1,032,000,000 ± tons; that the area under development originally contained 92,000,000 ± tons, out of which latter area 54,000,000 ± tons have been taken.

The Secretary presented, for M. John Marston, an illustrated set of formulæ for railroad turnouts and crossings.

Mr. John T. Boyd exhibited ribbons of phosphor-bronze with which he had experimented with a view to its use for tape lines, in mine, shop and other work, where the danger of breaking the ordinary steel lines is very great, and where the contact of tape with substances, in themselves injurious to it, renders frequent wiping, and consequent scouring off of the figures, necessary. The phosphor-bronze ribbon was found to be extremely tough, but, in addition to the difficulty in its manufacture into this shape, it was found that after it was bent at a sharp angle, it would not straighten out, thus decreasing the length of the line. As using the hammer to straighten it would increase the length, the experiment was not prosecuted further.

The Secretary presented a system of reduction tables which he had made to facilitate long and tedious multiplications and divisions.

AMERICAN SOCIETY OF CIVIL ENGINEERS.—The regular meeting of the Society was held Wednesday evening, March 21st, Vice-President Wm. H. Paine in the Chair, John Bogart, Secretary.

The death on March 8th was announced of James O. Morse, one of the earliest members, and who had been Secretary of the Society for 15 years, and Treasurer 21 years. An interesting collection of specimens of native wood was presented by John M. Goodwin, member of the Society. The subject of a continuance of tests of structural materials was considered. The Secretary made a statement of what had been done up to the present. Mr. O. Chanute, Chairman of a Committee on this subject, related the effort that had been made to secure larger appropriations from Congress; and the subject of the best method for conducting and continuing tests and of collating results so as to secure desirable information, was discussed

by Messrs. A. P. Boller, L. L. Buck, John Bogart, O. Chanute, T. C. Clark, Theodore Cooper, Charles E. Emery, Robert L. Harris, Charles Macdonald, Wm. H. Paine, and S. H. Shreve. Letters were read from General S. V. Benét, Chief of Ordnance, stating that the programme adopted for continuing tests of structural materials would be carried out on the Watertown Testing Machine to the extent of the very small amount appropriated by Congress, and the circular from the Chief of Ordnance, embracing that programme was also read.

A resolution was adopted to the effect that it was the sense of the meeting that a Special Committee should be appointed by the Board of Directors, to prepare and promote such a programme of tests of structural materials, as to secure the best results possible from the Watertown Arsenal experiments.

ENGINEERING NOTES.

ALEXANDRA DOCK.—Among important engineering works in England is the new dock at Hull, which has been designated the Alexandra Dock. Before describing its construction, however, it may be as well to notice briefly the accommodation Hull at present possesses in this respect. There is, of course, the old harbor or river Hull, which runs inland from the Humber and, winding through the town, has a very large number of quays and landing places along its banks. These, however, are only available for vessels of comparatively small tonnage and the waters of the river, moreover, are shallow and tidal. Situated along the bank of the Humber there are altogether seven wet docks, one graving dock, and two timber ponds. The wet docks, with their respective approximate water areas, are as follows :—Commencing from the western end of the range there is first the graving dock, which is 450ft. long by 50ft. wide, next the William Wright Dock of 7½ acres, adjoining which is the Albert Dock of 24½ acres, which is the largest existing dock in Hull. Next comes the North-Eastern Railway Dock of 2¾ acres after which, running inland in a curved line, are the Humber Dock of 7 acres, the Princess Dock of 6 acres, and the Queen's Dock of nearly 9 acres. Crossing the mouth of the river Hull we come to the Victoria Dock of 20 acres, beyond which are the timber ponds. The new Alexandra Dock, which will have a water area of 46½ acres, forms the eastern end of the range. Hull, therefore, at present, possesses 76½ acres of wet dock accommodation, besides which there is at the western extremity of the range a dock 10½ acres in extent now in course of construction. When all is completed Hull will possess about 134 acres of dock area, but with the great accession of trade which the increased accommodation may reasonably be expected to attract, it may be questioned whether a further extension may not be required before long. In fact, the new dock company have anticipated this by obtaining Parliamentary powers last Session to extend their accommodation to the eastward of their present works. With regard to the Alexandra Dock it will have a water area nearly twice as large as that of the Albert Dock. This latter dock is at present the only one practically available for the large vessels engaged in the Californian and East Indian trades, there being no warehouses of any extent available for grain at the Victoria Dock, and the other docks being too small for such vessels.

In order to construct the Alexandra Dock about 150 acres of land have been reclaimed from the Humber by embanking, so that the docks are really situated upon the foreshore of the river. Of these 150 acres about 100 will be occupied by the wet dock and two graving docks, with their quays, warehouses, roads, railways, and other adjuncts. It has a river frontage of 6,000ft., or more than a mile, with a depth to the rear of about 3,500ft. The dock itself has a total length of 2,300ft , and a width of 1,000ft., the area being, as we have already stated, 46½ acres. It is entered by a lock 550ft. in length and 85ft. in width, and having three pairs of gates and a caisson at the entrance. The lock is approached from the river through a trumpet-shaped entrance 360ft. wide, and having a wharf 300ft. long and constructed of timber on either side. These wharves are built on piles 60ft. long of creosoted timber. The water is shut out from the dock by a sea-bank one and a quarter mile in length, composed of 200,000 tons of chalk, and faced with Bramley Fall stone, with a slope of two to one on the sea face. The entrance to the lock works is at present protected by a cofferdam 500ft. long, constructed on a curve having a radius of 256ft. The dam is constructed of two rows of piles driven 6ft. apart, the intermediate space being filled in with puddled clay. There are 1,000 piles in the cofferdam of lengths varying from 50ft. to 60ft. There will be a depth of 34ft. of water over the sill of the dock at high water of ordinary spring tides, so that the largest class of shipping can be accommodated. In the dock, situated to the east of the entrance from the lock, will be two jetties, each 400ft. long and 80ft. wide. There will be a third jetty running out from the western side of the dock 450ft. long and 100ft. wide. The walls of the dock are 40ft. 6in. high from ground level, their depth below that point varying from 10ft. to 15ft. They are 20ft. wide at the base and 6ft. 9in. at the top. They are constructed of chalk rubble masonry faced with ashlar and finished with a granite coping. The jetties are also of masonry and of similar construction to the dock walls Generally the dock will be furnished with the necessary cranes, capstans, and other appliances, which, with the bridges, hoists, sluices, gates, and valves, will be worked by hydraulic power. This power, in fact, is already supplied and in use for driving a large amount of the machinery and appliances at present employed in the construction of the docks. The engine-house is situate at the north-eastern corner of the dock, and contains two double-cylinder high-pressure condensing engines, each of 150-horse power, by Sir W. Armstrong and Co. They take steam from six horizontal multitubular boilers of locomotive type,

working up to 80lb. per square inch. This develops in the accumulator a pressure of 700lb. per square inch for hydraulic purposes. The accumulator is 18in. in diameter, with a 35ft. stroke, the total weight on the water being 100 tons.

At the north-eastern angle of the main dock are two graving docks, the smaller of which, or No. 1, is 500ft. long by 60ft. wide on the floor, and 512ft. long by 81ft. wide on the top, with 19ft. 6in. of water on the sill. The walls of this dock, which are constructed of rubble masonry faced with Bramley Fall stone, are built up to within about 3ft. of the finished level. No. 2 dock is 550ft. long by 65ft. wide on tke floor, and 564ft. long by 89ft. wide on the top, with 21ft. 6in. of water on the sill. The walls are similarly constructed to those of No. 1, and have reached to within 7ft. of the finished level. The water will be pumped out of these docks by pumping engines, which will also be used for maintaining the water at the proper level in the main dock. The supply of water for this purpose will be drawn from an inland stream which drains the surrounding country and empties itself into the Humber The reason why this course is adopted in preference to replenishing the dock from the river is that the water from the latter source carries with it a large percentage of alluvial matter, which would deposit itself in the dock and necessitate frequent dredging. Thus two useful purposes are served, the drainage water is drawn off from the adjacent lands and a muddy deposit is prevented from accumulating in the dock.

The chief interest in the works at present centers upon the entrance lock, for constructing the walls and sluices of which two extensive trenches have been excavated. Each of these trenches is 680ft. long, 45ft. wide, and 45ft. deep, the sides being supported by an enormous mass of heavy timbering. The work is well forward in these trenches, and it is in them that the ceremony of setting the heel stones took place yesterday. Each of these stones consists of a block of Penrhyn granite from Cornwall, and each weighing eight tons. They will carry the gates which will close the entrance to the lock. The work of excavation in the dock is proceeding rapidly, a new feature being the introduction of hydraulic navvies, with which the contractors have superseded the steam navvy, which was extensively used by them in the construction of the Albert Dock at Millwall. It was found that if the steam navvy had more work to do than it fairly could it was strained, and soon got out of order. On the other hand, the hydraulic navvy cannot get strained, inasmuch as if set to do more work than it can accomplish, it simply stops dead short. The hydraulic navvy, moreover, is more economical to work, it requires two men only to control its movements, and performs an enormous amount of work. It is capable of removing from 600 to 700 cubic yards of soil per day, and is self-propelling, eating its way as it goes, and depositing the earth in tip wagons on either side as fast as they can be placed in position and removed. There is also another successful adaptation of hydraulic power for drawing the barrows of earth up an incline to bank from the bottom of the excavations, and lowering the empty ones. The chalk for the work is brought direct from some quarries, purchased by the contractors, the sand being taken from pits some 15 miles away.

Works of the extent of those under notice necessarily involve the services of several engineers. In the present instance, the engineer-in-chief for the dock is Mr. James Abernethy, Past President of the Institution of Civil Engineers, who is assisted by Messrs. Oldham and Bohn, of Hull, the acting engineers being Mr. A. Hurtzig and Mr. G. Abernethy.—*London Times.*

RAILWAY NOTES.

THE question of the necessity of introducing automatic couplings for railway purposes in England similar to those in use on some American, Colonial, and Continental railways, was brought before the members of the Manchester Association of Employers, Foremen and Draughtsmen, at their meeting in a paper read by Mr. J. Nasmith. Models of automatic combined buffers and couplings and of central automatic link couplings as used in the colonies were exhibited by Mr. Nasmith, who urged that an automatic coupling being proved to be mechanically practicable, the English companies ought to alter their present system without delay. The opinion of the meeting was, however, that an automatic coupling must be of such a character as to admit of gradual introduction to have any chance of adoption in that country. It must be designed so as to be applicable to existing stock without much alteration.

THE Channel Tunnel Railway Bill seeks to empower the Channel Tunnel Company, limited, to make the following railways in Kent in connection with the Channel tunnel, and for other purposes: Railway (1) 3 miles 1 furlong 2 chains, commencing in the parish of Temple Ewell by a junction with the London and Chatham Railway, and terminating in the parish of Guston, and at the Dover and Deal-road railway (2) 2 furlongs 3 chains, in Temple Ewell, commencing at the junction with the London, Chatham, and Dover railway and terminating by a junction with with railway (1); railway (3) 1 mile six furlongs 2 chains, commencing in Guston by a junction with No. 1 and terminating at low-water mark in the parish of West Cliffe. The capital of the company for the purposes of this act to be £750,000 divided into 37,500 shares of £20 each. The bill gives borrowing powers to the extent of £250,000, and the works are to be completed within ten years from the passing of the act.

A TABLE, showing the result of the working of the St. Gothard Railway from June to December, has been published. Excepting June, which was a broken month, August yielded the best returns—993,000f.—and December the lowest—685,000. On the other hand, while the working expenses were the lowest in August and September, not exceeding 300,000f., they rose in December, owing to the necessity of keeping the line clear of snow, to 426,000f. The excess of receipts over expenses

during the period in question makes a total in round numbers of three and a-half million francs. The goods traffic shows a steady increase, the receipts from this source being 500,000f. in December, against 380,000f. in August. The falling off in the winter returns arises exclusively from the diminution in the number of passengers. In August 117,000 passengers travelled over the line; in December only 50,000. The number of passengers carried by the company since the opening of the line last June is 601,000.

ORDNANCE AND NAVAL.

THE ITALIAN NAVAL REVIEW OF 1882.—An official account of the naval review held by King Humbert at Spezzia on October 17, and of the conclusions drawn from it, has lately been published. His Majesty examined the 45-centim. breechloading cannon destined for the Italia, and the heavy artillery on the fortifications, reviewed the marines and soldiers of the garrisons, and witnessed several experiments with submarine mines and torpedoes, including a sham fight between the Duilio and Dandolo, and four first-class torpedo boats, which formed the escort of the royal launch. The points considered to be established by these experiments are the following: A stationary vessel would be certainly struck by a torpedo, discharged from a torpedo boat approaching her, within 220 yards distance. There is great probability that during an actual engagement torpedo boats would be able to come unperceived within this distance of an ironclad, in spite either of daylight or the electric light, under the cover of the smoke of her own guns. First-class torpedo boats will form, therefore, one of the principal elements in the coast defence of the future, and will render the operations of bombardment both dangerous and difficult. It is very probable that mistakes in the working of the torpedo-discharging apparatus may occur during the excitement of an engagement, and consequently it is highly desirable that the men should be well practiced in the duties. The first-class torpedo boats at present in use, although behaving well at sea, cannot develop their full speed if it is rough. Submarine torpedoes, laid to defend the anchorage of ships, entrance of harbors, &c., can be innocuously exploded by a system of counter-mines, dropped rapidly by swift steamers, and discharged without loss of time. The war material furnished for the defence of the Gulf of Spezzia is in excellent condition, and well adapted to its purpose. The artillery, electric lights, and torpedoes of the Duilio and Dandolo are entirely satisfactory, as is also the working of their engines.

BOOK NOTICES.

PUBLICATIONS RECEIVED.

FROM MR. JAMES FORREST we have received the following papers of the Institution of Civil Engineers:

The Horizontal Thrust of a Mass of Sand. By George Howard Darwin, F. R. S.

The River Indus. By Charles Swaine Fahey, M. I. C. E.

Modern Corn-Milling. By Professor Cesare Saldini

Various Methods of Determining Dimensions. By Dr. James Weyrauch.

The Capacity of Storage Reservoirs. By W. Ripple.

REPORT OF THE NEW YORK STATE SURVEY FOR 1881. By James T. Gardiner.

MONTHLY WEATHER REVIEW FOR FEBRUARY. By Maj-Gen. W. B. Hazen.

TWENTY YEARS WITH THE INDICATOR. By Thomas Pray, Jr., C. E. Vol. I. Boston: Journal of Commerce Publishing Co.

This is a treatise on the construction and use of the Indicator, with examples from actual use exhibited by diagram.

As an instruction book for the learner, this seems by reason of the large number of examples to be well adapted.

HYDRAULIC MANUAL. By Louis D'A. Jackson. London: Crosby, Lockwood & Co.

A summary of the text of this book is briefly given as follows:

I. Explanation of Principles and Formulas.

II. Field Operations and Gauging, with brief accounts of modes adopted.

III. Miscellaneous Paragraphs on Hydraulics.

About one-third of the book is filled with the tables commonly used for hydraulic calculations.

The treatise is designed as a guide to the survey of rivers and canals rather than to the minutiæ of supply and drainage.

THE FORESTS OF ENGLAND. By J. C. Brown, LL.D.

This is an attractive historical sketch of the English forests, going back to the time of the Roman invasion.

Forests that have been submerged since the beginning of historic times come in for their share of attention.

"The Devastation of Forests," and "Forest Legislation" are topics upon which the writer is a leading authority, and upon which he discourses at some length in this volume.

DETAILS OF MACHINERY. By Francis Campin. London: Crosby, Lockwood & Co.

This is the latest addition to the Weale's Rudimentary Series. The plan of the book is somewhat novel, as the elementary parts of machinery are separately considered.

Arithmetical computations of strains and dimensions abound throughout the book, and everywhere elaborate rules are made to take the place of formulas. No algebraic expressions are employed, not even a plus sign.

The work will prove serviceable for the class of artisans for whom it is specially designed, as it furnishes them with a collection of useful rules and illustrative examples, using only arithmetical computations.

THE GREAT PYRAMID. By Richard A. Proctor. New York: R. Worthington.

To those who have read the previous essays of Mr. Proctor relating to the Pyramid, this

will prove an interesting supplement. All who have regarded the theory of Prof. Smyth or of Abbé Moigns at all favorably, should read this new essay carefully. It does not need, however, an acquaintance with previous writings to read this book with pleasure and profit. It is written in the author's best vein.

The book also contains the following essays: The Origin of the Week, The Sabbath of the Jews, Astronomy, and the Jewish Festivals, and The History of Sunday.

MISCELLANEOUS.

BEFORE the Paris Academy of Sciences a paper was recently read on "Hydraulic Silica, and on the part it plays in the Hardening of Hydraulic Compound," by M. Landrin. The pure silica obtained by decomposing a solution of silicate of potash with an acid, and repeatedly washing and drying at a dark red heat, he names hydraulic silica, and he considers it the cause of the final hardening of hydraulic mortars. The aluminate of lime cannot concur in this effect, because of solubility, but at the moment of immersion it facilitates the intimate union of the hydraulic elements, hinders water from penetrating the mass of mortar, and so aids the slow reciprocal action of the lime and hydraulic silica.

OWING to its greater strength, phosphor-bronze is used sometimes instead of copper for conducting electricity, since much smaller wires possesses the necessary strength. The resistance offered by phosphor-bronze is considerably greater than that of copper, so that while it answers well for telephone wire, it is not adapted to long telegraph lines. Herr L. Weiller, of Angouleme, has recently alloyed copper with silicon instead of phosphorus, and made a silico-bronze, the conductivity of which, the *Scientific American* says, is twice that of phosphor-bronze, while its strength is not less, and hence seems well adapted to electric conductors. The relative strengths of copper, silico-bronze, and phosphor-bronze, are said to be as 28, 70, and 90; conductivity as 100, 61, and 80.

AT a recent meeting of the Paris Academy of Sciences a paper was read describing some researches on the relative corrosion of cast iron, steel, and soft iron, by M. Gruner. Various plates, suspended in a frame by their four corners, were immersed simultaneously in water acidulated with 0.5 per cent. of sulpauric acid, or sea water, or were simply exposed in moist air of a terrace. In moist air, chromate steels were corroded most rapidly, and tungsten steels less than carbon steel. Cast iron, even, with manganese, is oxidized less than steel and soft iron, and white specular iron less than grey cast iron. Sea water, on the other hand, attacks cast iron more than steel, and with special energy white specular iron. Tempered steel is less attacked than the same steel annealed, soft steel less than manganese steel or chromate steel, &c. Acidulated water, like sea water, dissolves grey cast iron more rapidly than steel, but not white specular iron; the grey impure cast iron is most strongly attacked. These results agree with the complete experiments on the subject by Mallet in 1843.

MANGANESE BRONZE.—The use of this alloy as a material for screw propeller blades is rapidly extending. The first run made by the Alaska between Queenstown and New York in less than seven days was immediately after her steel blades had been replaced by others of manganese bronze. Nearly every clipper ship afloat is fitted, or is being fitted, with them; the Galia, the Orient, the Austral, the Normandie, the Stirling Castle, and many other less known names, figure in the list issued by the Manganese Bronze and Brass Company, Limited, together with one or two war vessels belonging to our own and other governments. The qualities that render the metal peculiarly suitable for propellers, are its great strength and its non-liability to corrosion. Experiments carried out at the works of Messrs. Maudslay, Sons, and Field, show that the bronze has a transverse strength about double that of gun-metal, and up to the elastic limit, double that of steel. In the tests that were made, the steel took a permanent set of .01 in. with a strain of 10 cwt., as against 20 cwt. required to give the manganese bronze a similar set. In manufacturing steel blades an extra thickness has to be provided, to allow of the rapid pitting that takes place on the back, and even with this provision the life of the blade rarely exceeds three years. But with bronze blades no such allowance need be made; they can be cast of the thickness to give them the requisite strength, and put in their places without undergoing the annealing which often results in the distortion of steel blades. As regards cost, steel blades can be bought for 50*l.* to 60*l.* per ton, while if constructed of bronze their price is 125*l.* to 135*l.*, or if allowance be made for the difference of weight, about double that of steel. Assuming the steel blades to require renewal every three years, it is calculated that by the time the second set are fitted the expense has run up to the price of bronze blades, which are stated to be good for the life of the vessel, and then to have considerable value as old metal.

THE steamship "Tartar," lately built for the Union Steamship Company, has the following dimensions. Length, 392ft.; breadth, 47ft.; depth, 33ft. 6in. Estimated gross tonnage, 4,359, with a displacement of 8,000 tons. The "Tartar" is divided into thirty watertight compartments, and has a double bottom constructed on the cellular system, extending the whole length of the ship, and capable of containing 500 tons of water as ballast. The ship is built to meet all the requirements of the Admiralty, and will be placed on their list of vessels for use in time of war. Her outside plating is doubled, to ensure greater strength amidships. The dining saloon, on the upper deck forward, is 62ft. long, and of great height. Above the saloon is a large music room and ladies' boudoir, opening on the promenade deck, which is nearly 180ft. long. The engines have cylinders 50in. and 94in. diameter, with a stroke of 60in., and are 650-horse power nominal.